

This Is Time in Graph Drawing

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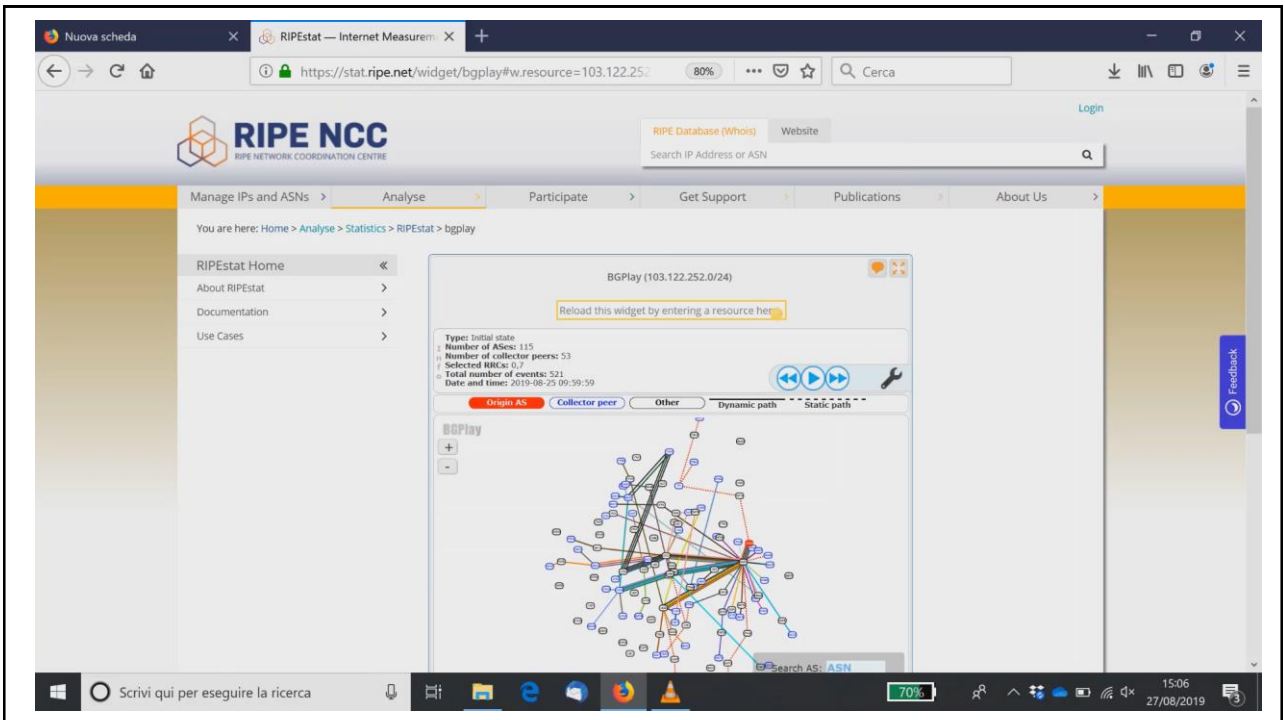
27th International Symposium on Graph Drawing and Network Visualization – GD2019
Průhonice/Prague, September 17-20, 2019

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BGPlay

- My starting point (time): Bgplay
 - gdb Mariani Patrignani Pizzonia, Bgplay: A System for Visualizing the Interdomain Routing Evolution, GD 2003
 - part of the RIPE Stat service
 - its Web page is visited an average of more than five thousands times per day
 - also available at RouteViews, BGPStream, and Isolario

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The concept of time in Graph Drawing

- One of the main challenges for Graph Drawing is the relationship between drawings and time
 - Show the temporal evolution of the visualized graphs
- About 40 papers on the subject in the GD Conferences
- Main GD fields related to *time*
 - dynamic algorithms, streaming, animation, time reconciliation, storyline, morphs, human perception

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The point of view of the InfoVis community

- Visualization metaphor
 - Node-link or matrices
- Span of knowledge on data
 - Offline if, at any time instant, future data are known and can be used to compose the drawing
 - Online otherwise
- Representation of time
 - Animation or timeline
- Mental map preservation
- Modeling of transitions

Beck, Burch, Diehl, Weiskopf, The state of the art in visualizing dynamic graphs, Eurovis 2014

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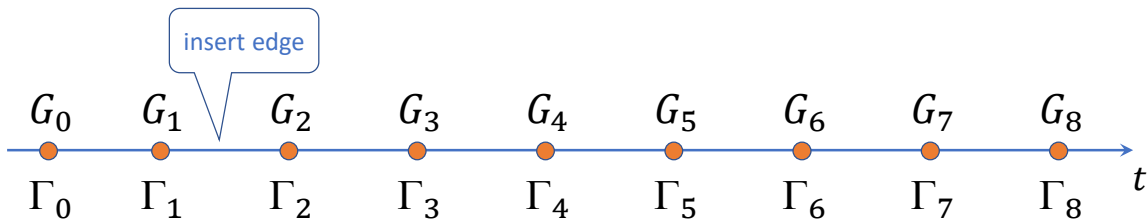
Purpose and limits of this talk

- Assess the maturity level of GD on the topic
- Mainly focusing on
 - Personal experience
 - Combinatorial algorithms
 - Straight-line drawings
 - Networking applications
- General methods
- Open problems

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Graph stories and drawing stories

- A *graph story* is a sequence $G_0 \cdots G_n$ of graphs, where G_{i+1} is obtained from G_i by applying an *update operation*
- A *drawing story* $\Gamma_0 \cdots \Gamma_n$ of a graph story $G_0 \cdots G_n$ is a sequence of drawings such that Γ_i is a drawing of G_i



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Type of graph stories and of drawing stories

- A graph story is a *tree story* or a *forest story* or a *planar graph story* or if all G_i 's are trees or forests or planar graphs or
- A drawing story is *planar* or *straight-line* or if all its drawings are planar or straight-line or

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Dynamic Algorithms

	Animation	Timeline
Offline		
Online	Dynamic Algorithms	

The drawing story is *not* known in advance
 Rely on an *implicit* representation of the drawing

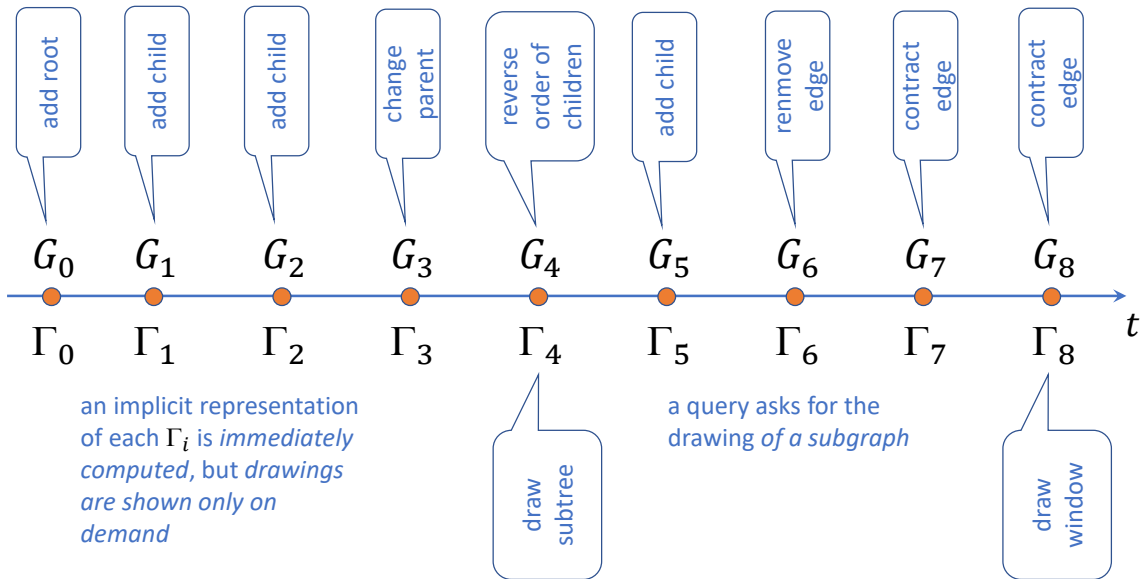
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A Dynamic GD Problem

- Type of the graph story and type of the drawing story
- Repertory of update operations
 - E.g. insertion of vertices and edges
- Queries on the current drawing
 - E.g. draw subgraph, draw window
- Dynamic drawing predicate
 - Similarity properties of consecutive drawings

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Dynamic GD – a drawing story of a forest story



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Selected contributions

- Cohen gdb Tamassia Tollis Bertolazzi, A framework for dynamic graph drawing, SoCG 1992
- Cohen gdb Tamassia Tollis, Dynamic graph drawings: Trees, series-parallel digraphs, and planar st-digraphs, SICOMP 1995
- Papakostas Tollis, Incremental orthogonal graph drawing in three dimensions, GD 1997
- Bachl, Semi-dynamic orthogonal drawings of planar graphs, GD 2002
- Plus several papers that focus on the topology of the drawing, e.g. dynamic planarity testing

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Some results

Graph story	Drawing story	Memory	Update time	Draw subgraph time	Draw window time	Overall area
Forest of rooted trees	Upward planar straight-line grid	$O(n)$	$O(\log n)$	$O(k + \log n)$ Subtree	$O(k \log n)$	$O(n^2)$
Series-parallel digraphs	Upward planar straight-line grid	$O(n)$	$O(\log n)$	$O(k + \log n)$ Series-parallel subgraph	$O(k \log^2 n)$	$O(n^2)$
Planar <i>st</i> -digraphs	Upward planar <i>polyline</i> grid	$O(n)$	$O(\log n)$	$O(k \log n)$	-	$O(n^2)$

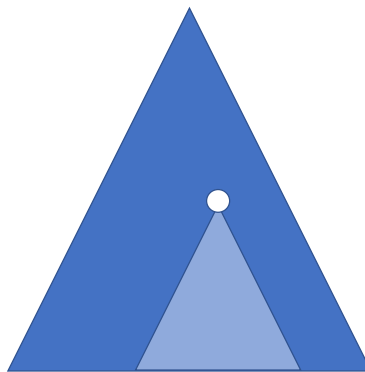
n is the total number of vertices and k is the number of vertices to be drawn

targets: a time that is: (1) sublinear in n for updates and (2) linear in the number of objects to be drawn and sublinear in n for drawing queries

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Dynamic GD – methods

- Maintain a tree decomposition of the graph equipped with variables representing the features of the drawing
- The decomposition is implemented using dynamic trees



The position of a node in the drawing depends on several features of the subtrees rooted at that node

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Dynamic GD – problems

- Currently, the tree drawings that are maintained use the x -coordinates of an in-order traversal; is it possible to maintain Rengold-Tilford-type drawings?
- Maintain a straight-line grid drawing of a planar graph (e.g. a Schnyder-wood)

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Streaming

	Animation	Timeline
Offline		
Online		Streaming

The drawing story is *not* known in advance
 When an *object* (vertex/edge) arrives, it is explicitly drawn, its placement cannot (almost) be altered, objects have a given *persistence*

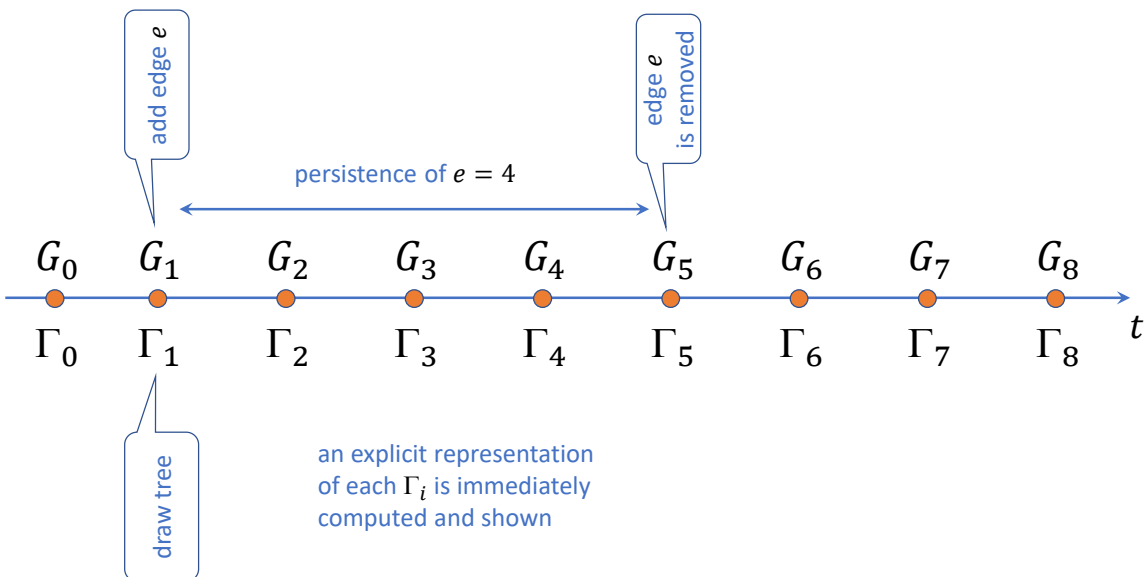
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A Streaming GD Problem

- A source produces a sequence of n objects (vertices/edges) that are immediately and explicitly drawn
- The *persistence* of an object specifies how much time it remains in the drawing; typically all objects have the same persistence (also called *window size*) $k \ll n$
- Once an object is drawn, its placement *cannot be altered* until it is removed (k instants after it was produced)
- All drawings should fit an area that is a function of k

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Streaming GD – a drawing story of a tree story



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Selected contributions

- Binucci Brandes gdb Didimo Gaertler Palladino Patrignani Symvonis Zweig, Drawing trees in a streaming model, GD 2009
- Nguyen Eades Hong, Streameb: Stream edge bundling, GD 2012
- Goodrich Pszona, Streamed graph drawing and the file maintenance problem, GD 2013
- Crnovrsanin Chu Ma, An Incremental Layout Method for Visualizing Online Dynamic Graphs, GD2015

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Some Results

Graph story	Drawing story	Objects	Memory	Update time	Number of points in convex position
Degree d Trees (Eulerian Tour only)	planar straight-line, circular layout	edges	$O(k)$	$O(k)$	$\left\lceil \frac{k}{2} \right\rceil (d - 1) + k + 1$
Trees (Eulerian Tour only)	planar straight-line, circular layout	edges	$O(k)$	$O(k)$	$2k - 1$

k is the persistence (size of the window)

target: *reuse* the area

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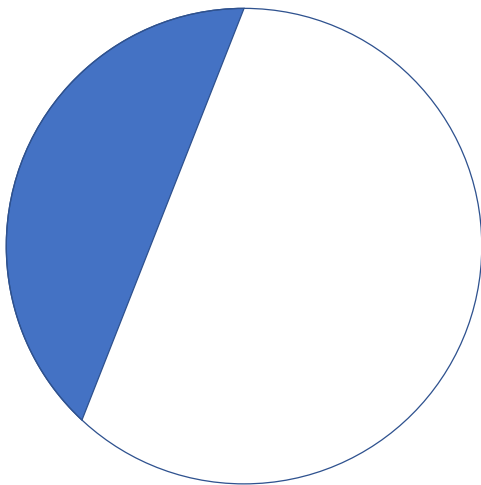
Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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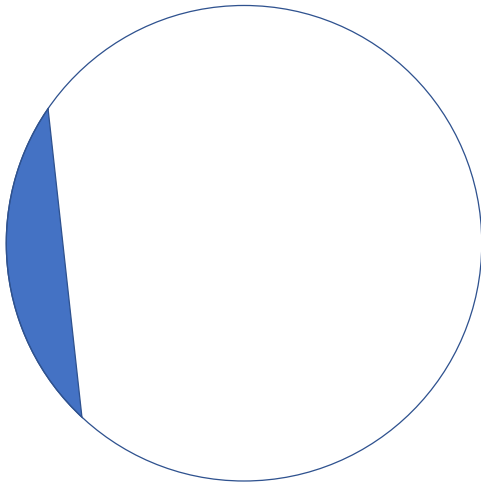
Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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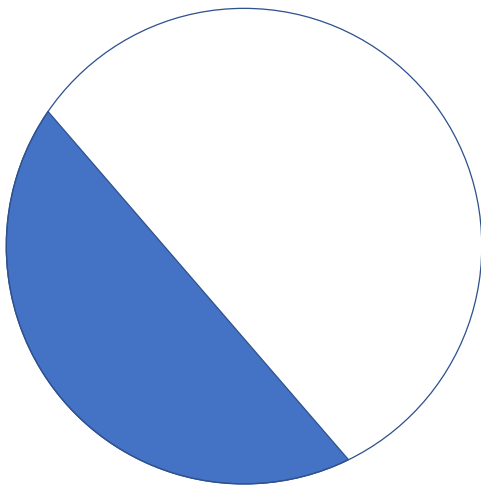
Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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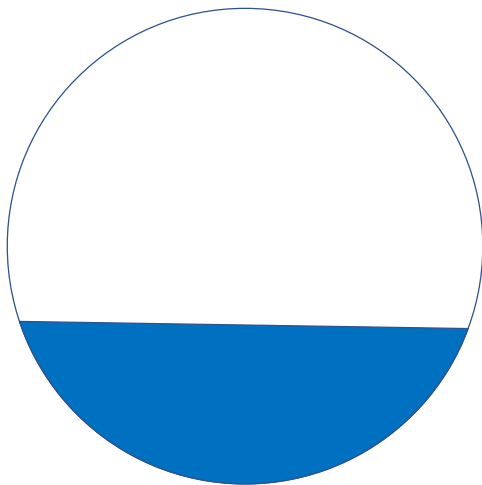
Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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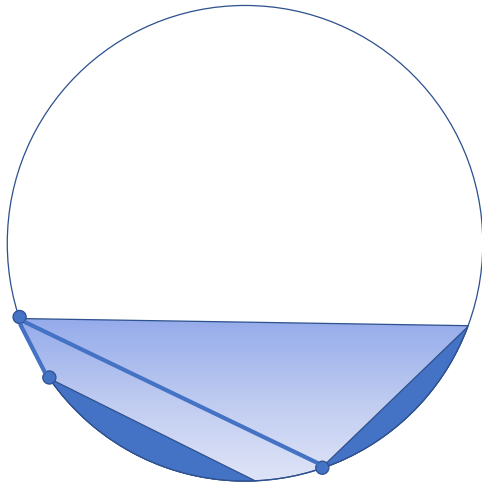
Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

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Methods – the art of reusing the space

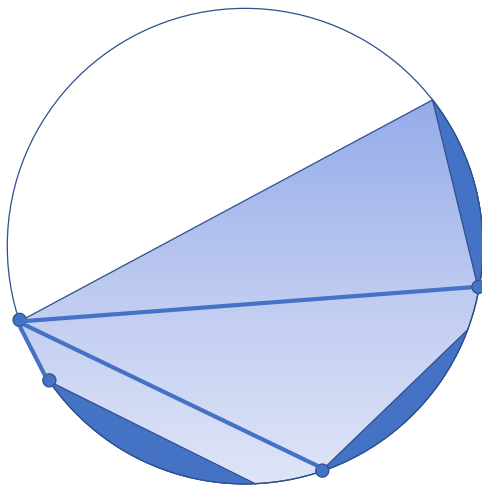


Evolution of the drawing during the Eulerian tour of the tree

Eulerian tour = DFS + explicit backtrack edges

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Methods – the art of reusing the space

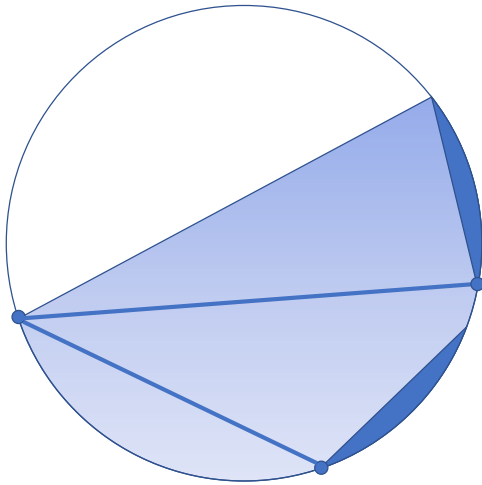


Evolution of the drawing during the Eulerian tour of the tree

Eulerian tour = DFS + explicit backtrack edges

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Methods – the art of reusing the space

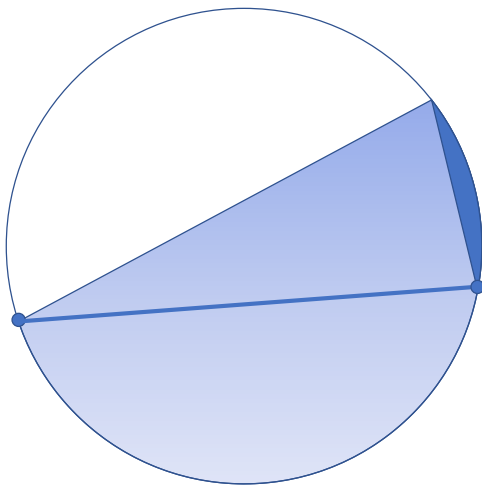


Evolution of the drawing during the Eulerian tour of the tree

Eulerian tour = DFS + explicit backtrack edges

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Methods – the art of reusing the space

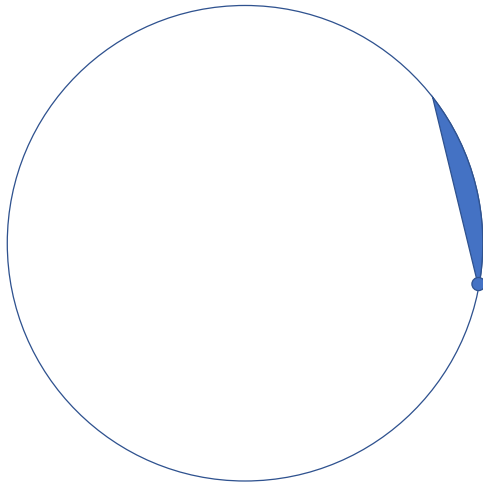


Evolution of the drawing during the Eulerian tour of the tree

Eulerian tour = DFS + explicit backtrack edges

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Methods – the art of reusing the space



Evolution of the drawing during the Eulerian tour of the tree

Eulerian tour = DFS + explicit backtrack edges

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If we use dynamic graph drawing for streaming

Graph story	Drawing story	Objects	Memory	Update time	Number of points in convex position (streaming)	Area
Trees (Eulerian Tour)	planar straight-line, circular layout	edges	$O(k)$ $O(k)$	$O(k)$ $O(k)$	$2k - 1$	$O(k^3)$ $O(k^2)$

k is the persistence (size of the window)

Dynamic graph drawing bound

So what? In dynamic graph drawing the objects *move*, even of a linear amount of space for just one update

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Streaming – problems

- Non-tree stories
- Non-Eulerian-tour visit
- Explore the area-movement tradeoff, following the developments in Goodrich Pszona GD2013

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Offline - animation

	Animation	Timeline
Offline		
Online		

All the updates are known in advance
 The graphs of the resulting sequence are drawn one-after-the-other so that the mental map is preserved

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Selected contributions

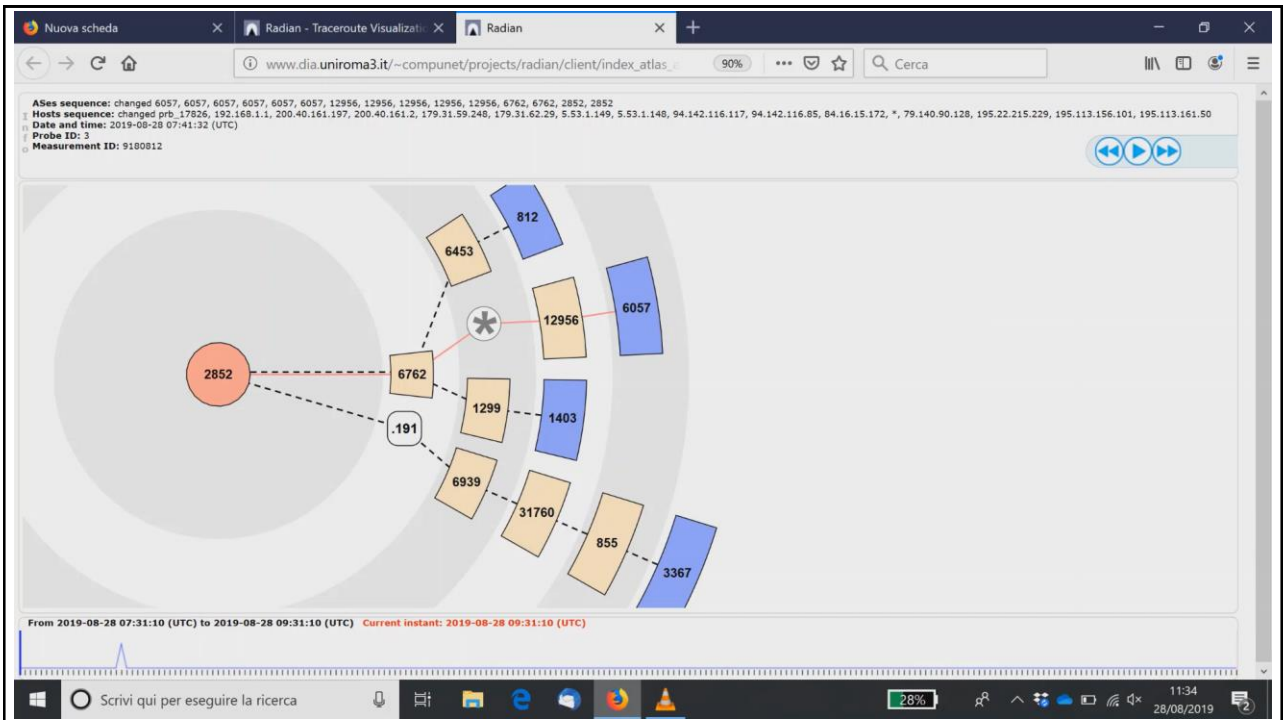
- Brandes Wagner, Dynamic grid embedding with few bends and changes, ISAAC 1998
- Diehl Görg, Graphs, they are changing, GD 2002
- Erten Harding Kobourov Wampler Yee, Graphael: Graph animations with evolving layouts, GD 2003
- Brandes Fleischer Puppe, Dynamic spectral layout with an application to small worlds, JGAA 2007
- Da Lozzo Rutter, Planarity of streamed graphs, CIAC 2015
- Skambath Tantau, Offline drawing of dynamic trees: Algorithmics and document integration, GD 2016
- *Borrazzo Da Lozzo Frati Patrignani, Graph Stories in Small Area, GD 2019*

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Radian

- Candela Di Bartolomeo gdb Squarcella, Radian: Visual Exploration of Traceroutes, TVCG 2018
- Application of clustered planarity techniques
- available at
<http://www.dia.uniroma3.it/~compunet/projects/radian/>

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Radian Methods

- **Augment clusters**
 - Add vertices and edges into clusters so that every path traversing a cluster c has the same number of vertices in c
- **Assign layers**
 - Compute a layering for all vertices
- **PQ-tree layout**
 - Use a PQ-tree to order vertices along the layers so that (1) vertices on a layer and belonging to the same cluster are consecutive and (2) the number of edge crossings is low
 - The PQ-tree is initialized with a spanning tree of the graph and is incrementally updated with the remaining edges, which induce ordering constraints

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Offline animation – problems

- In some problems it is difficult or impossible to maintain certain invariants of the drawing
 - *Temporal cuts* can be introduced (see Skambath Tantau 2016) when some invariants are violated and the drawing is “reset”
 - Look for drawing stories with few temporal cuts

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Storyline & C.

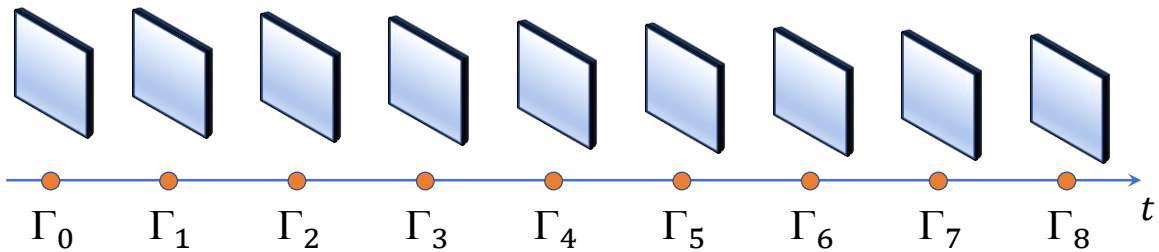
	Animation	Timeline
Offline		Storyline & C.
Online		

One Cartesian coordinate represents time

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Timeline representation – an example

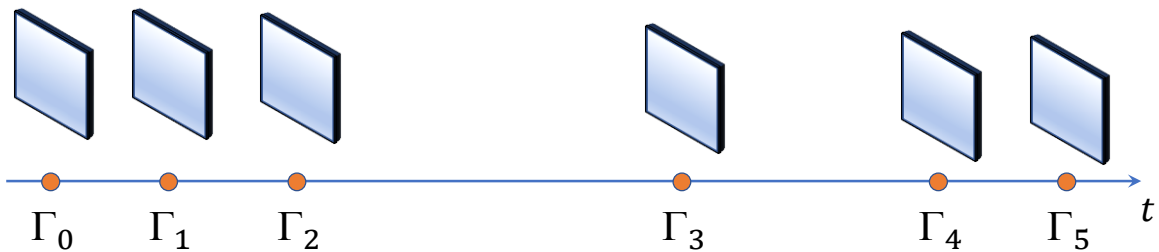
- All the Γ_i are *together* in the drawing



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Timeline representation – deformation of time

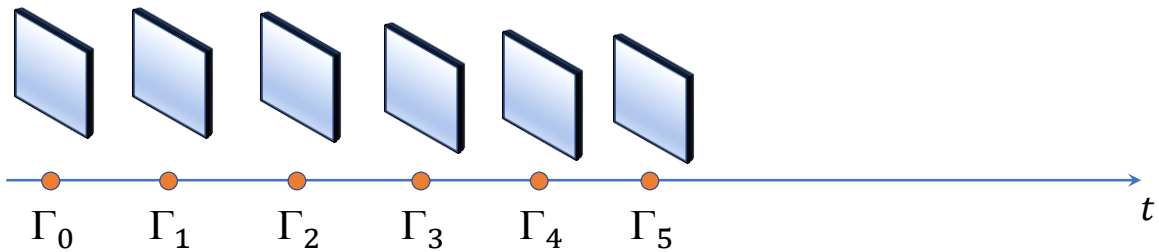
Wang Archambault Haleem Moeller Wu Qu, Nonuniform Timeslicing of Dynamic Graphs Based on Visual Complexity, VIS 2019



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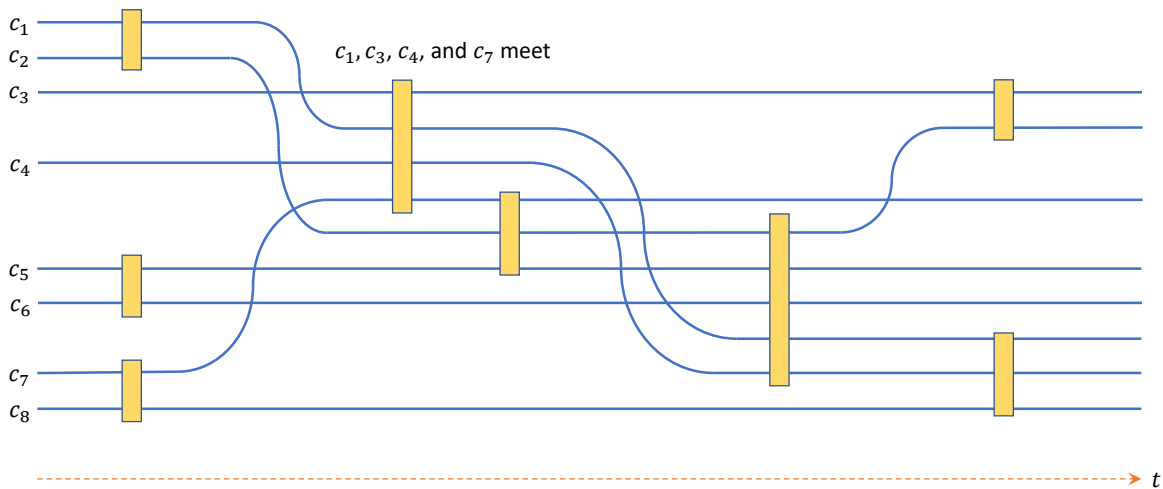
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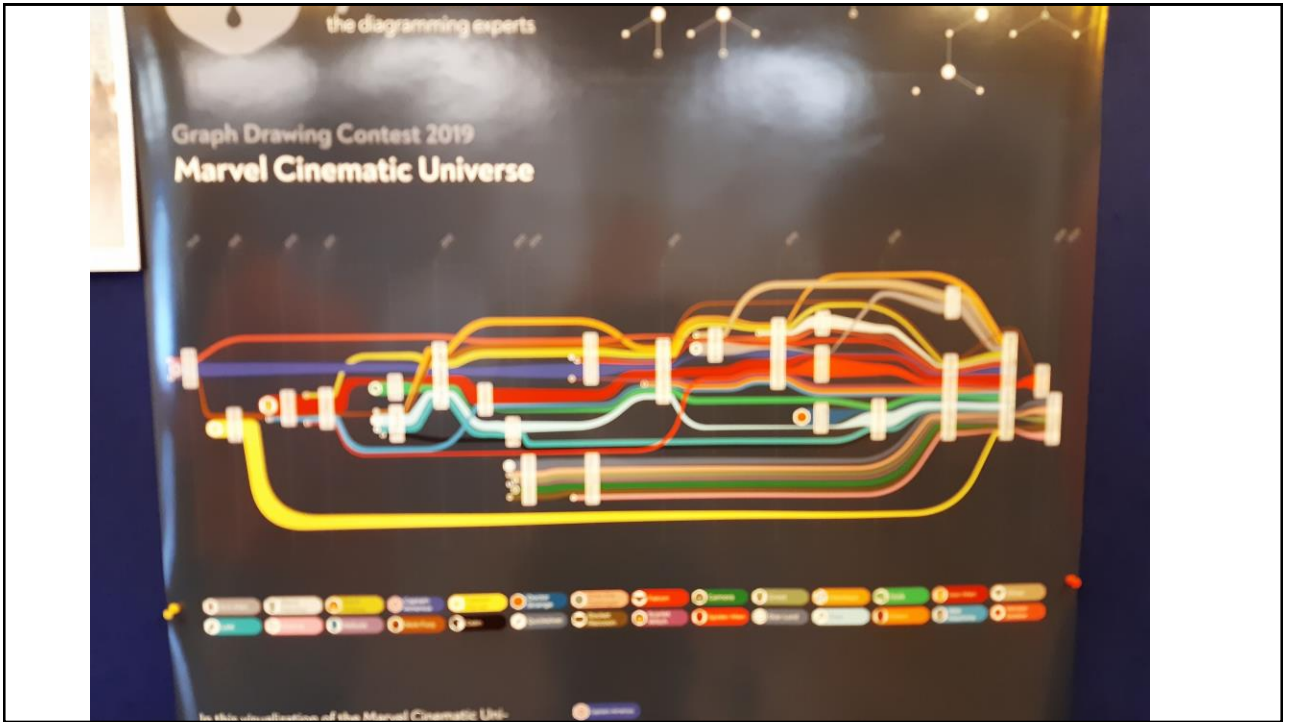


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Storyline - abstraction of the structure of a narrative



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Goals and Methods

- Minimize crossings between characters
- Minimize crossings between bundles of characters (block crossings)
- Exact algorithms
- Integer Linear Programming
- SAT solvers

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Selected contributions

- Tanahashi Ma, Design Considerations for Optimizing Storyline Visualizations, TVCG 2012
- Kostitsyna Nöllenburg Polishchuk Schulz Strash, On minimizing crossings in storyline visualizations, GD 2015
- van Dijk Fink Fischer Lipp Markfelder Ravsky Suri Wolff, Block crossings in storyline visualizations, GD 2016
- Gronemann Jünger Liers Mambelli, Crossing minimization in storyline visualization, GD 2016
- van Dijk Lipp Markfelder Wolff, Computing storyline visualizations with few block crossings, GD 2017

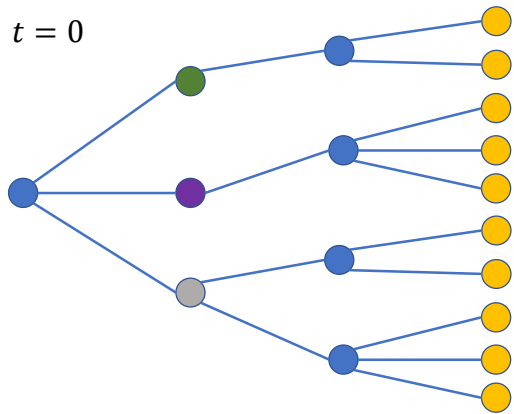
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Upstream visibility

- Candela gdb Marzialetti, Upstream Visibility: A Multi-View Routing Visualization, VINCI 2018

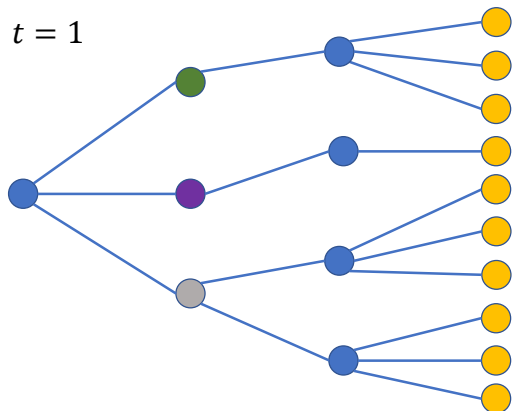
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A streamgraph paradigm for routing visualization



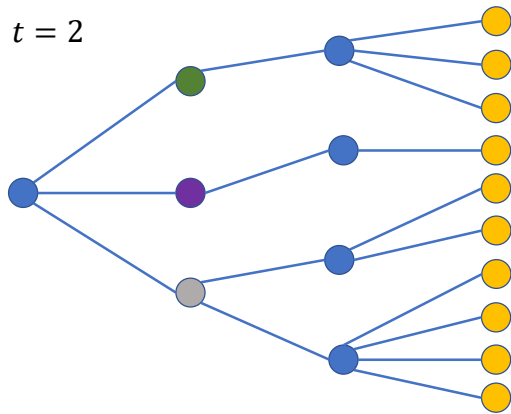
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A streamgraph paradigm for routing visualization



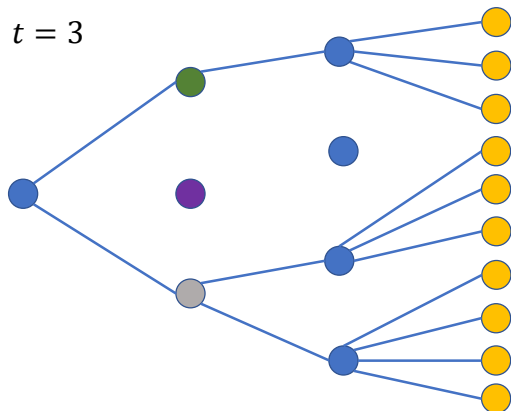
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A streamgraph paradigm for routing visualization



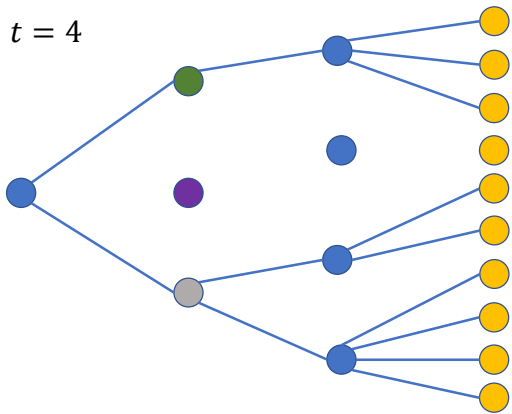
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A streamgraph paradigm for routing visualization



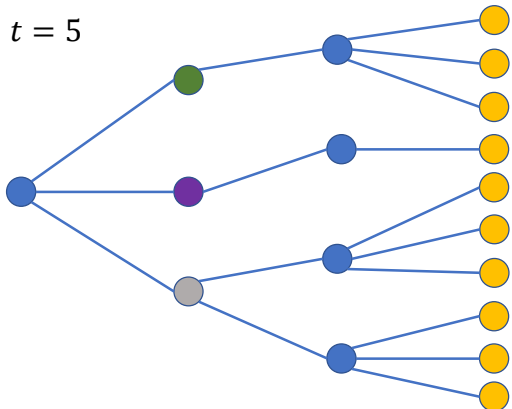
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A streamgraph paradigm for routing visualization



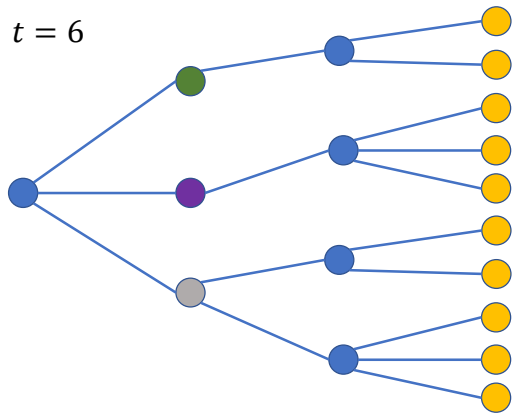
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A streamgraph paradigm for routing visualization



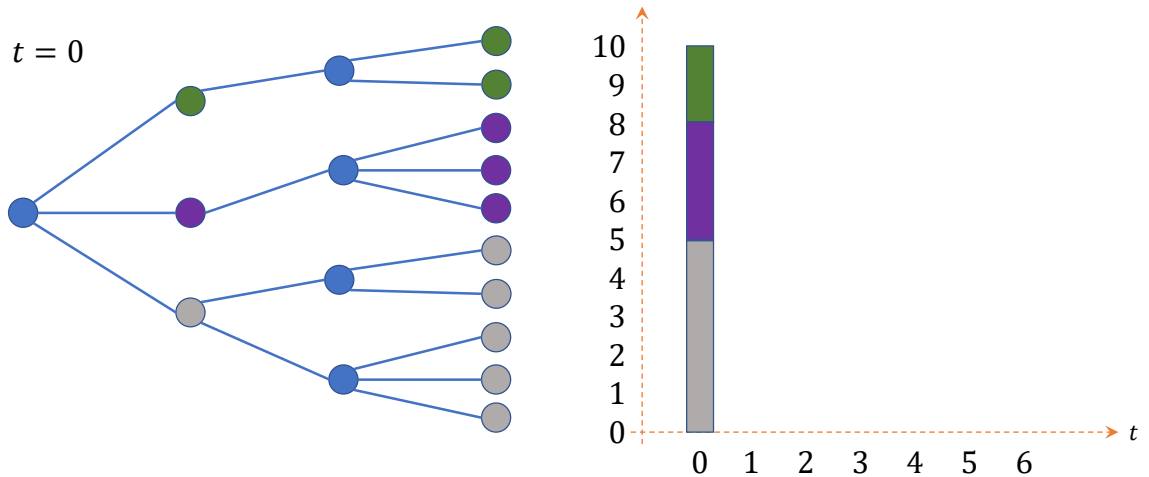
54

A streamgraph paradigm for routing visualization



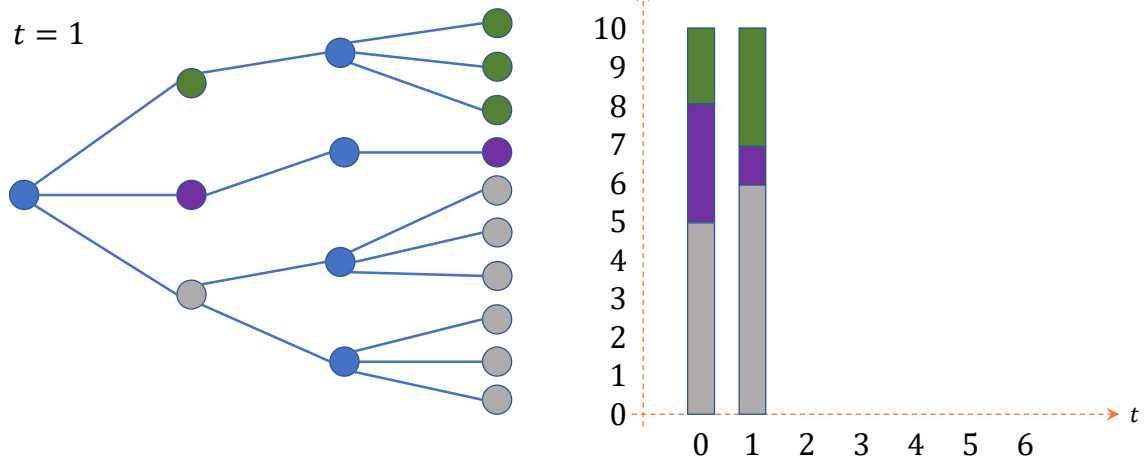
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A streamgraph paradigm for routing visualization



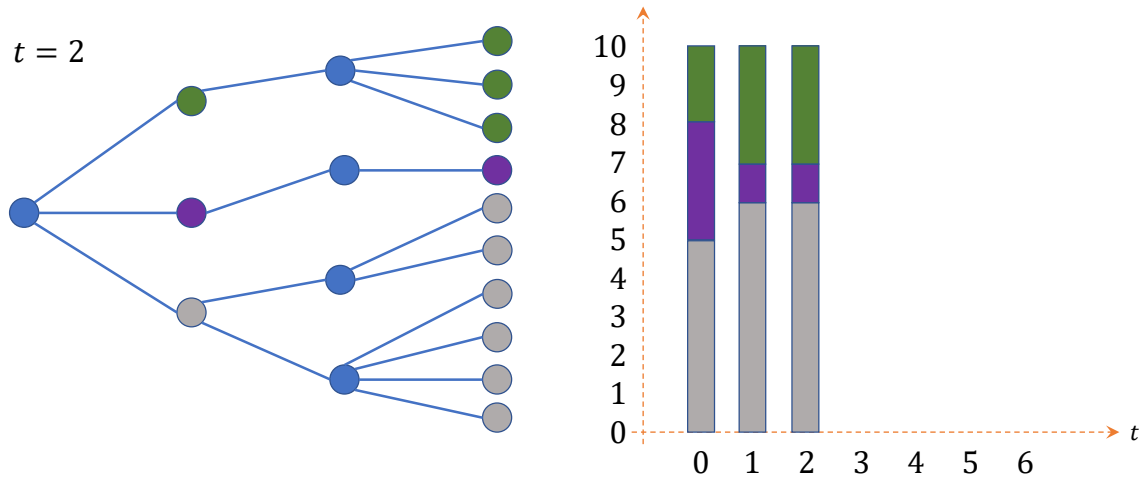
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A streamgraph paradigm for routing visualization



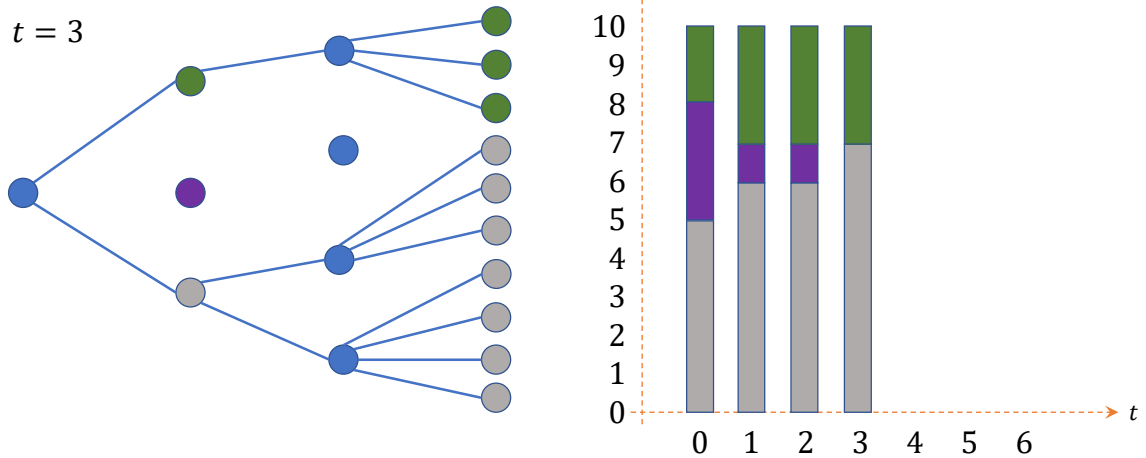
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A streamgraph paradigm for routing visualization



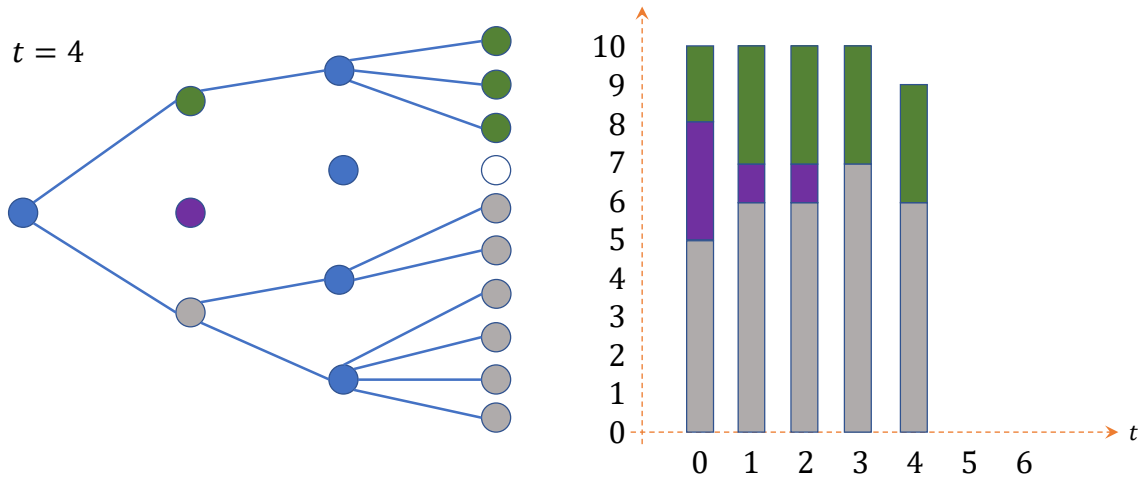
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A streamgraph paradigm for routing visualization



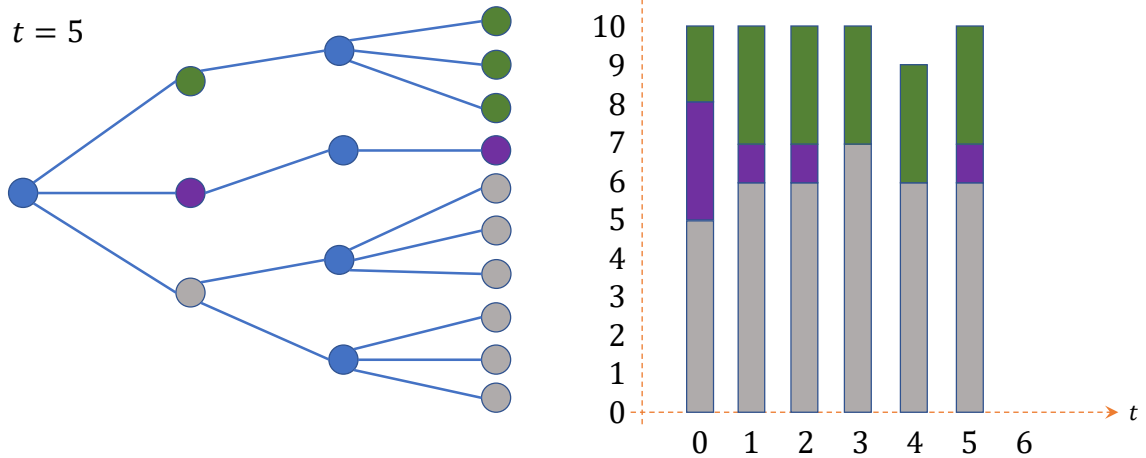
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A streamgraph paradigm for routing visualization



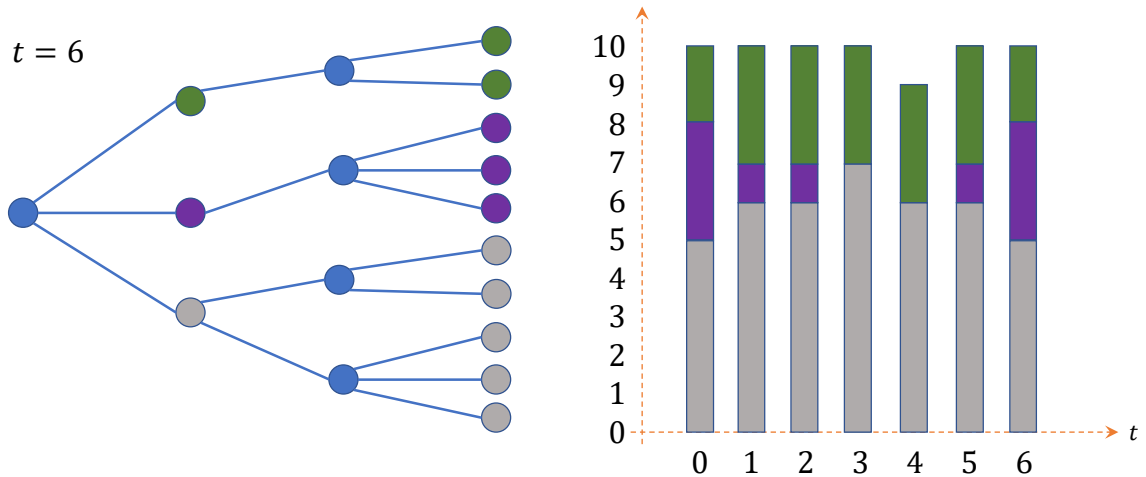
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A streamgraph paradigm for routing visualization



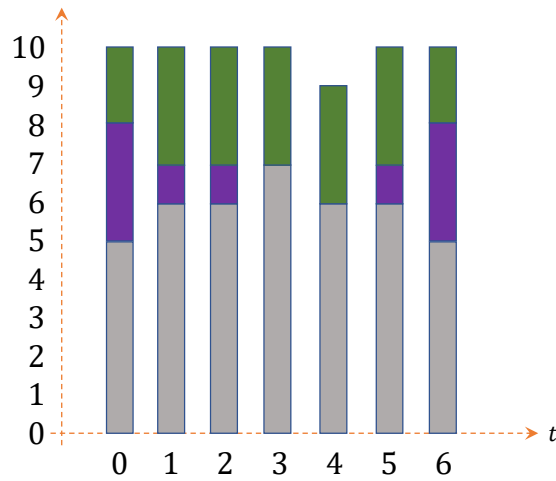
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A streamgraph paradigm for routing visualization



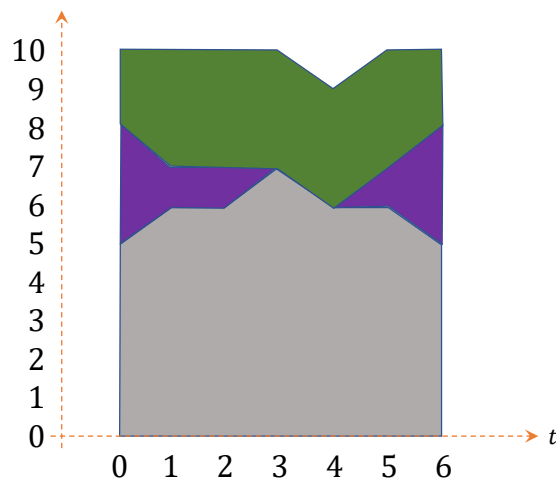
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A streamgraph paradigm for routing visualization



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A routing streamgraph

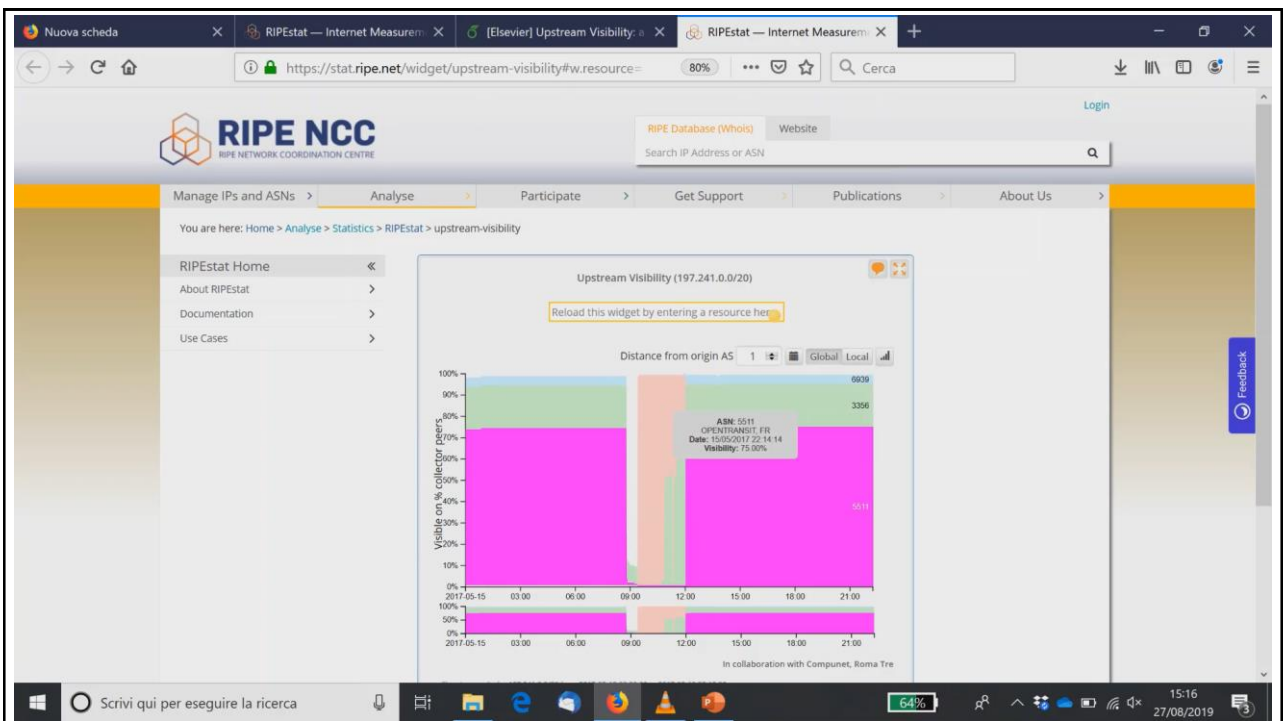


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Upstream Visibility clip

- service available at RIPE Stat

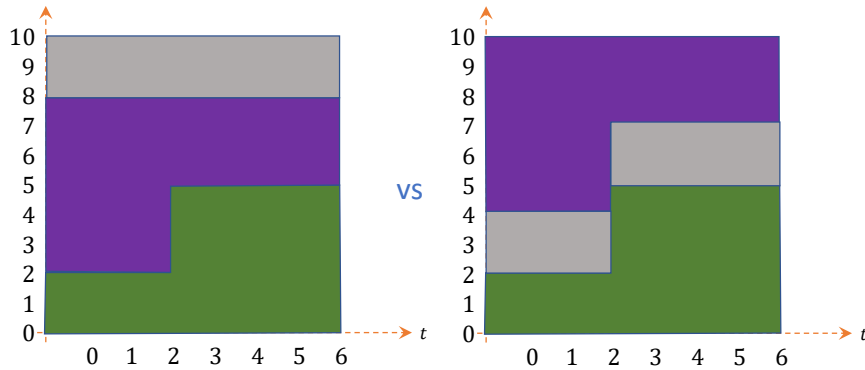
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Streamgraph – problems

- Find an ordering of the curves so that each colored region is «smooth»



See also: Di Bartolomeo Hu, [There is more to streamgraphs than movies: Better aesthetics via ordering and lassoing](#), Computer Graphics Forum 2016

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Timeline representations – problems

- How to visualize a graph story according two or more clocks?
 - Physical quantity clock
 - Event clock
 -
- Lotker, [The tale of two clocks](#), IEEE/ACM ASONAM 2016
 - using the drift between clocks is useful to understand the dynamics in social networks

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Morphs

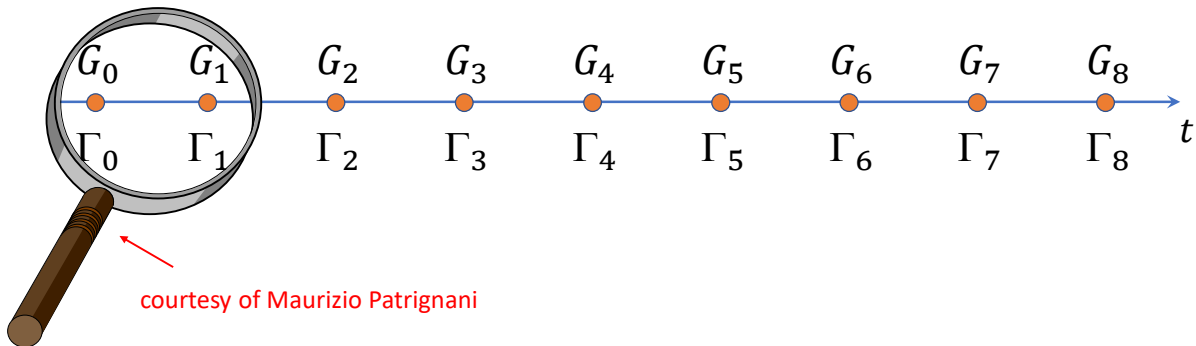
	Animation	Timeline
Offline	Morphs	
Online		

How to transform Γ_i into Γ_{i+1}

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A closer look at one of the steps

- Focus on Γ_0 and Γ_1
- In general they are different
- How to transform (morph) one into the other?



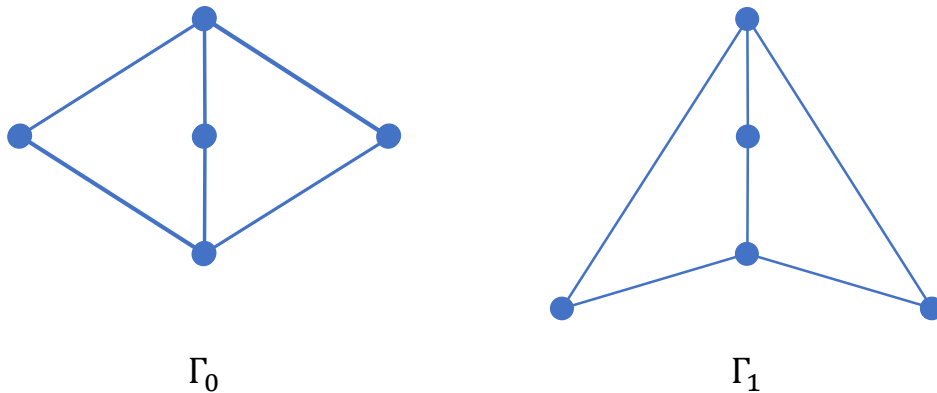
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The problem of morphing graph drawings

- input: two drawings Γ_0 and Γ_1 of the same graph G
- A *morph* between Γ_0 and Γ_1 is a continuously changing family of drawings of G indexed by time $t \in [0,1]$, such that the drawing at time $t = 0$ is Γ_0 and the drawing at time $t = 1$ is Γ_1

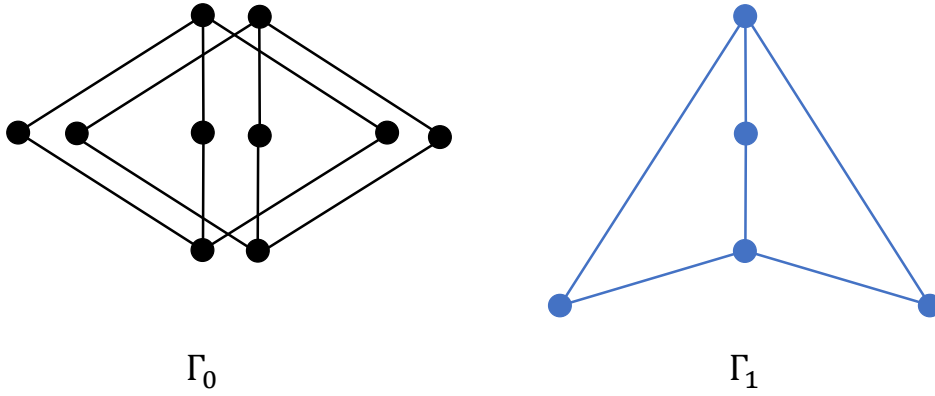
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A morph



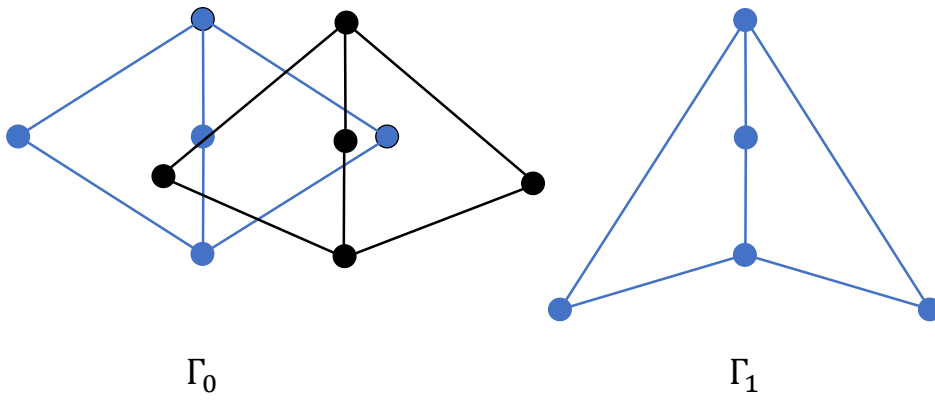
72

A morph



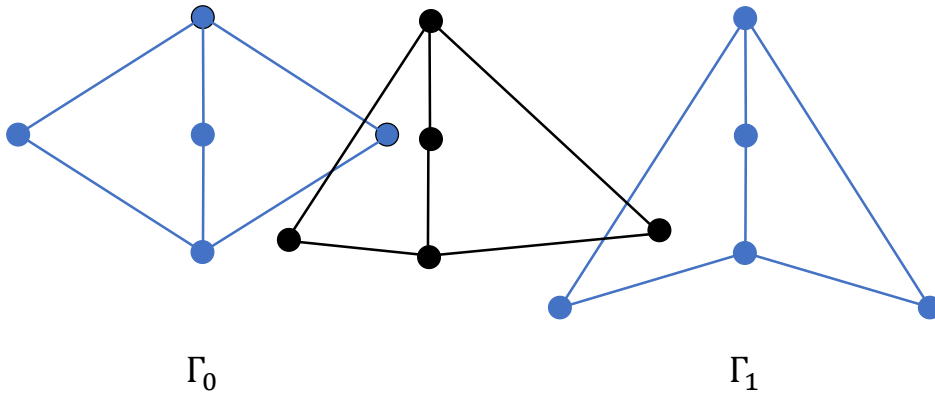
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A morph



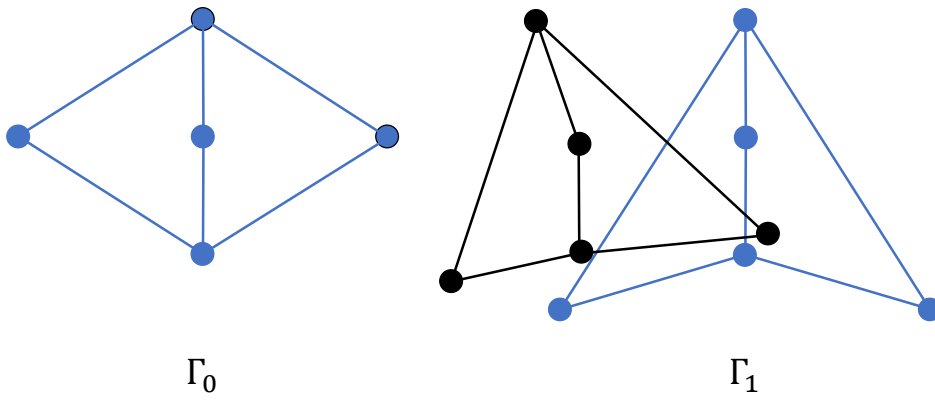
74

A morph



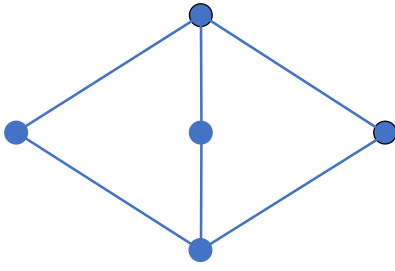
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A morph

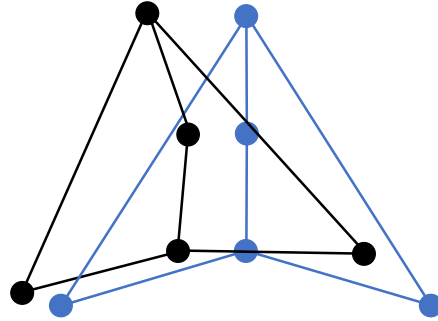


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A morph



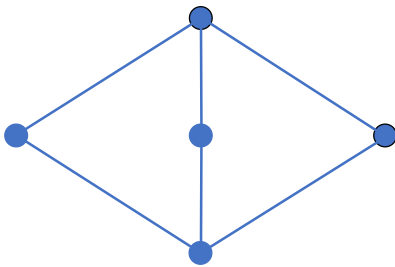
Γ_0



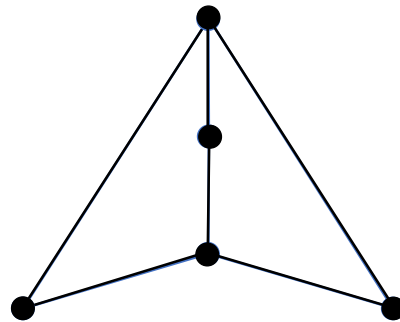
Γ_1

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A morph



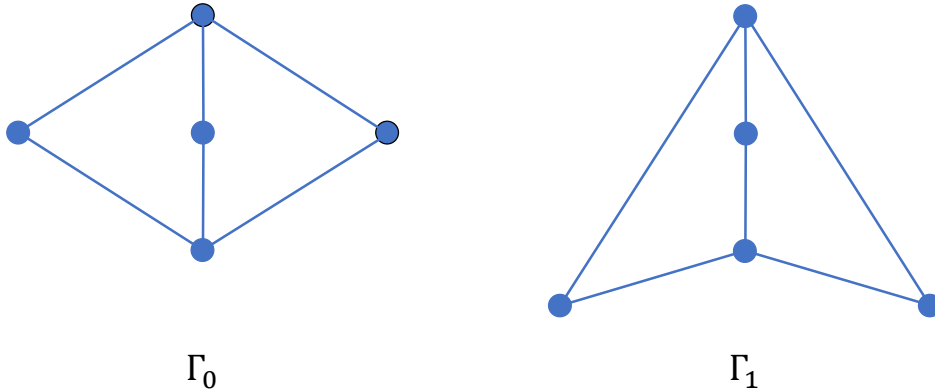
Γ_0



Γ_1

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A morph



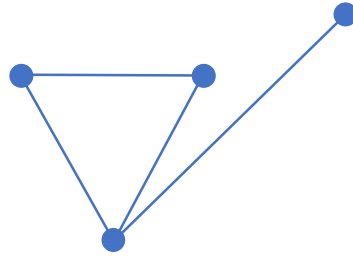
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Morphing preserving one or more properties

- suppose that both Γ_0 and Γ_1 have a certain geometric property, e.g.
 - they are planar drawings
 - they are straight-line drawings
 - their edges are polygonal lines composed of horizontal and vertical segments
 - their faces are convex polygons
 -
- it is interesting that all the drawings of the morph *preserve* that property

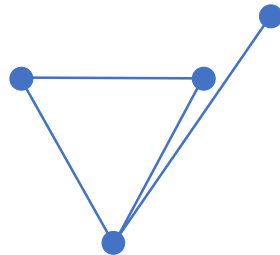
80

A morph that does not preserve planarity



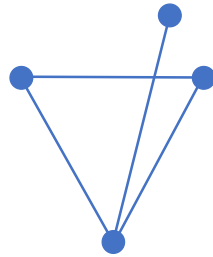
81

A morph that does not preserve planarity



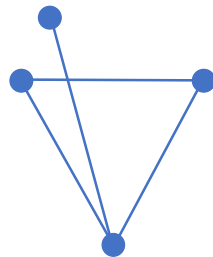
82

A morph that does not preserve planarity



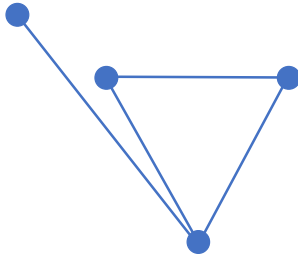
83

A morph that does not preserve planarity



84

A morph that does not preserve planarity

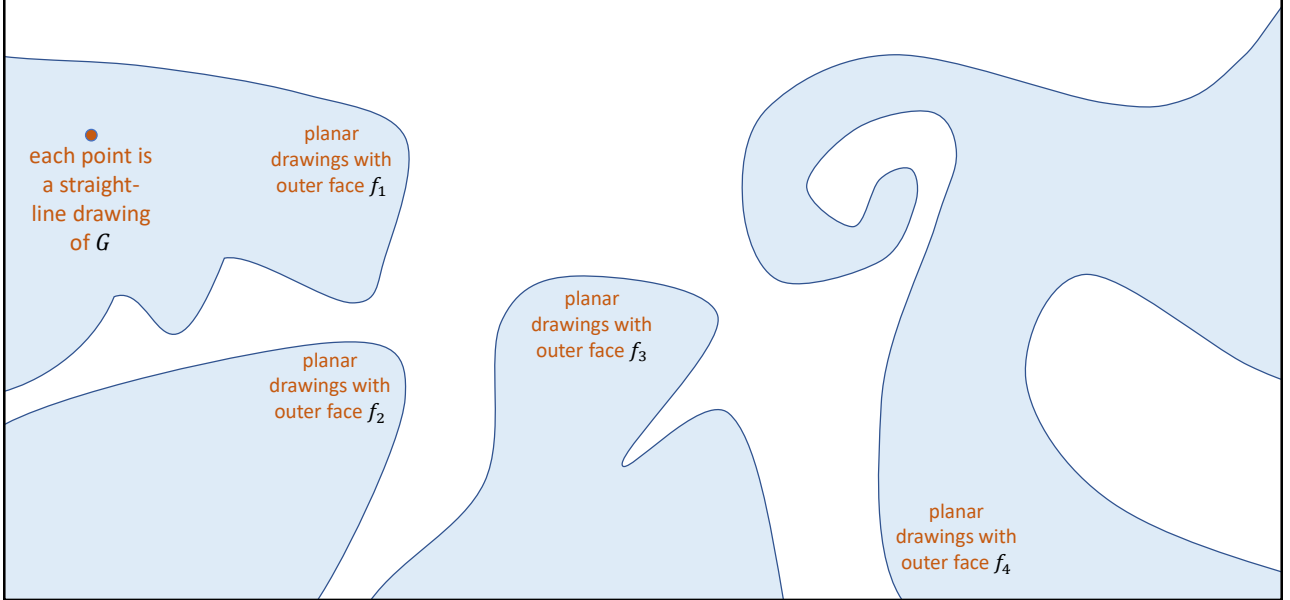


85

morphs
straight-line drawings of
triangulations

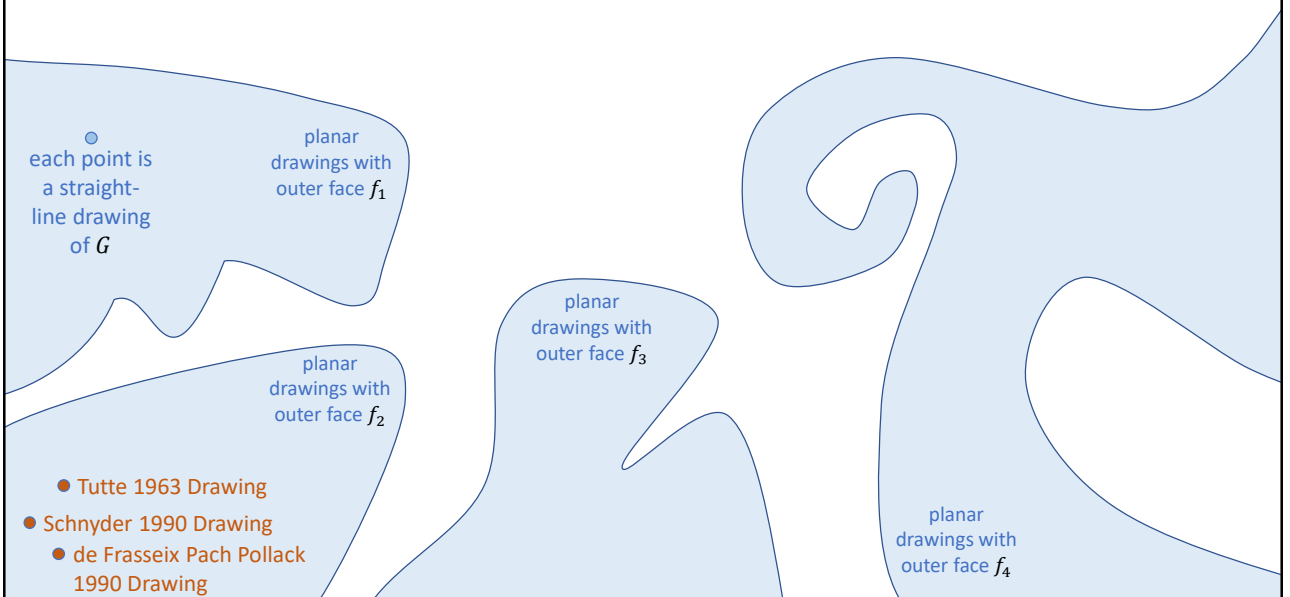
86

Straight-line drawings of a triangulation G with n vertices in R^{2n}



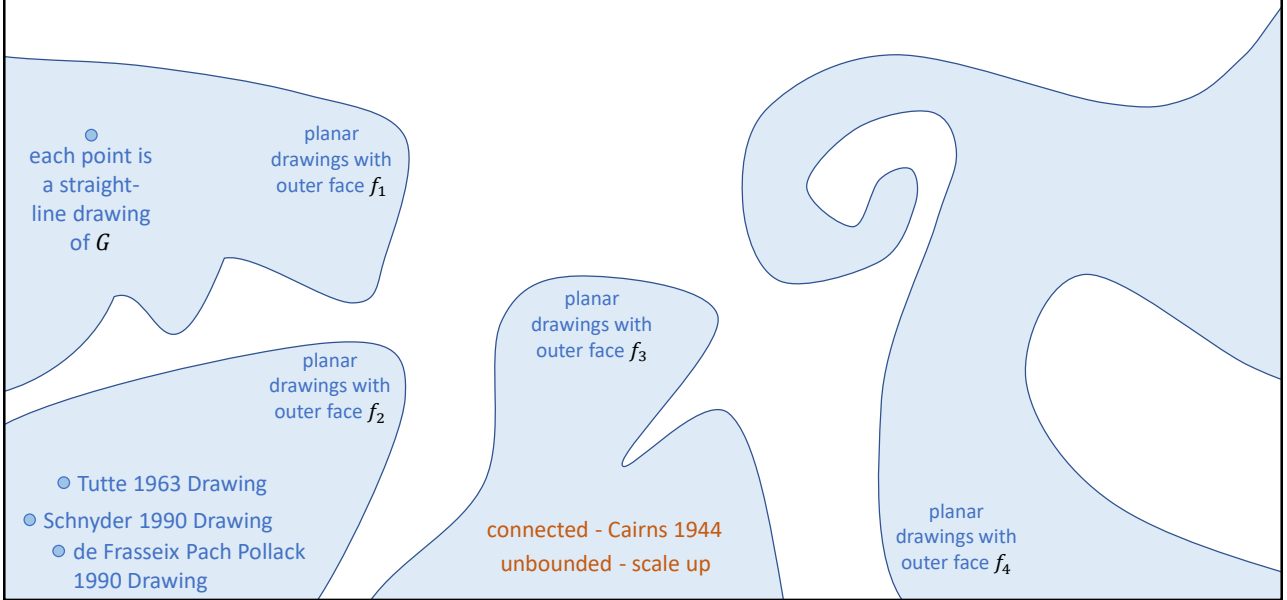
87

Straight-line drawings of a triangulation G with n vertices in R^{2n}



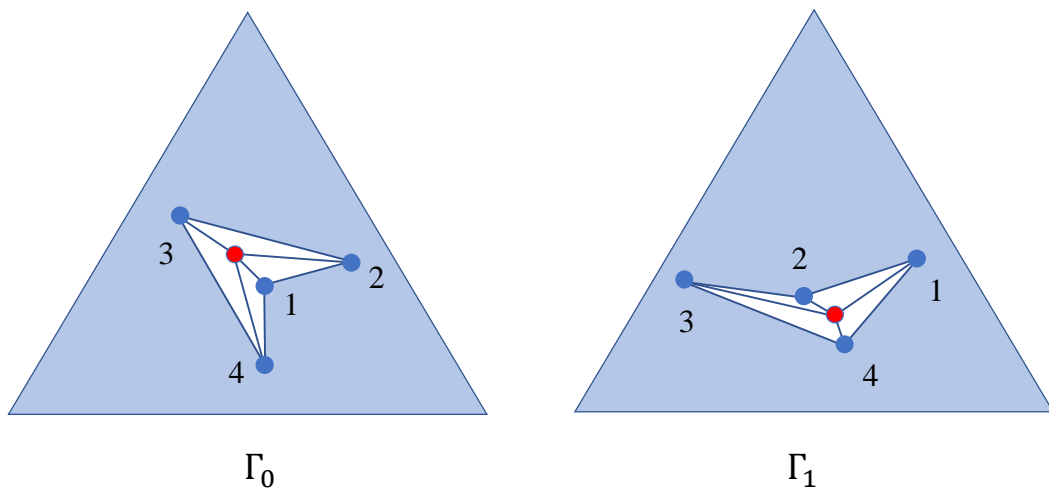
88

Straight-line drawings of a triangulation G with n vertices in R^{2n}



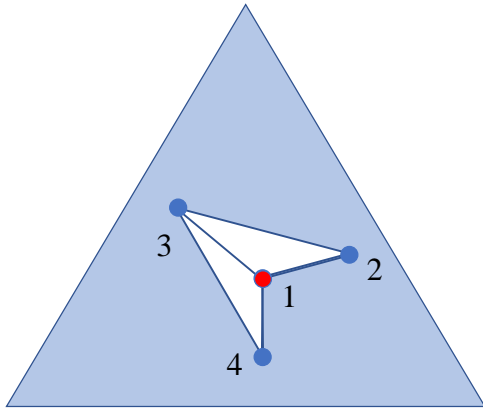
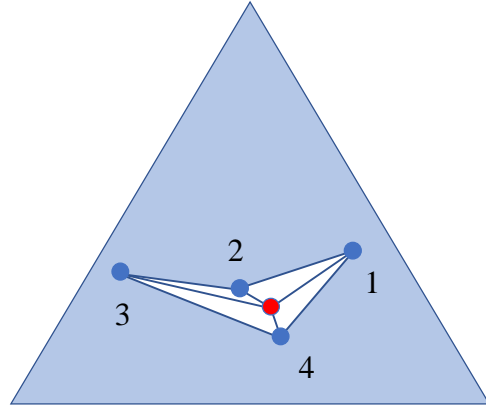
89

Cairn's double recursion approach



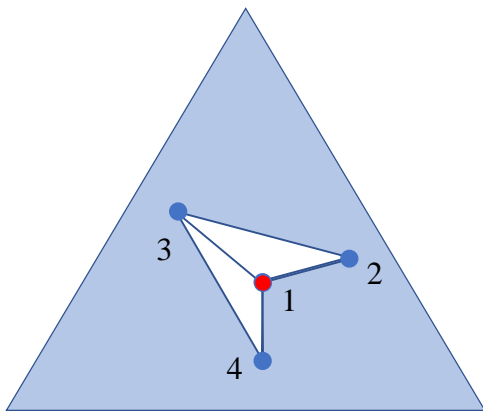
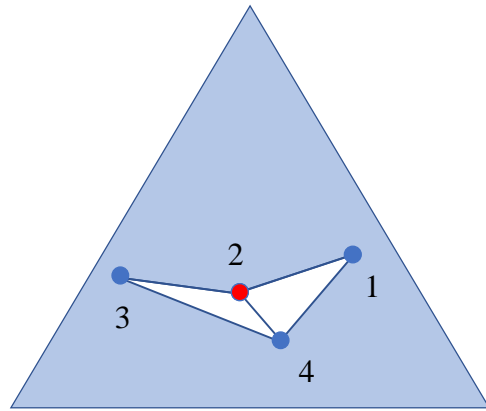
90

Cairn's double recursion approach


 Γ_0

 Γ_1

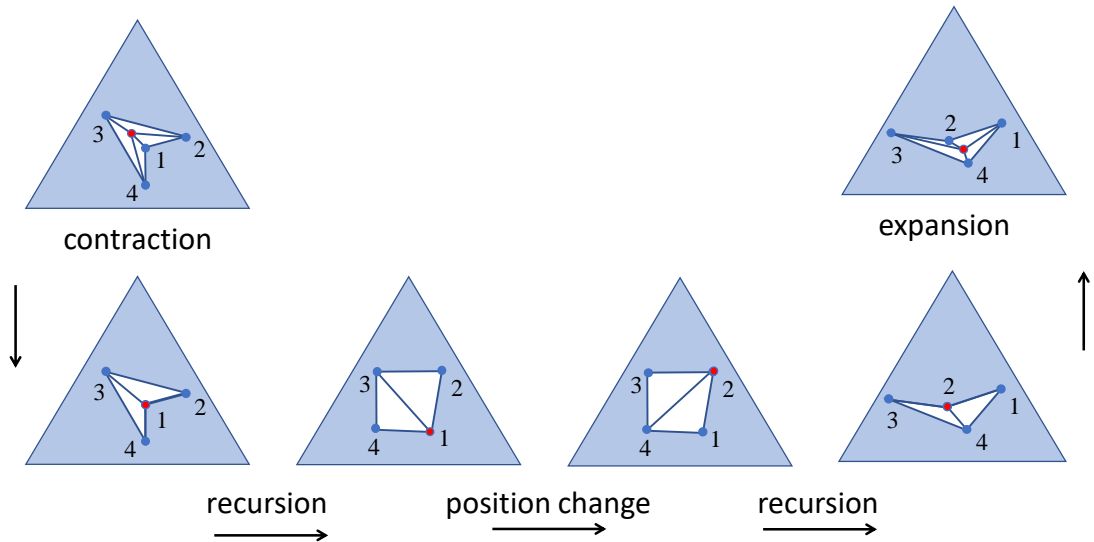
91

Cairn's double recursion approach


 Γ_0

 Γ_1

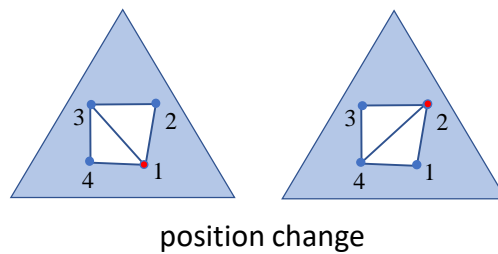
92

Cairn's double recursion approach



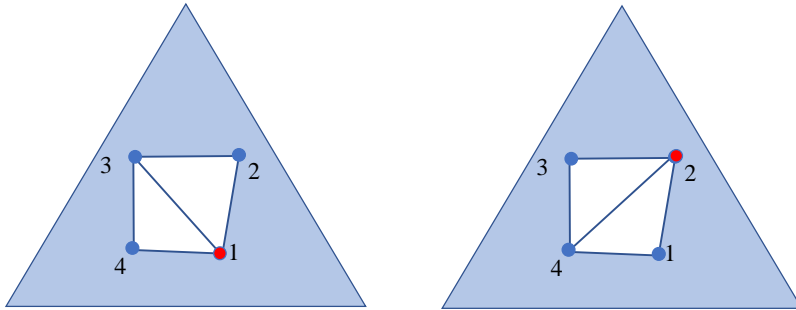
93

A closer look at the position change



94

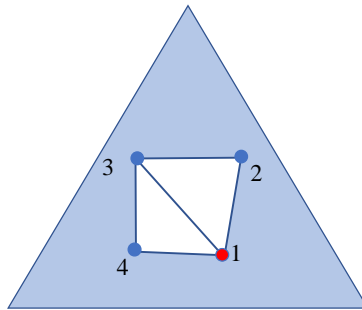
A closer look at the position change



position change

95

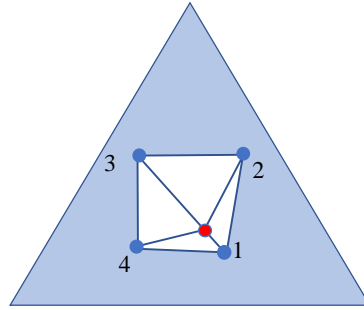
A closer look at the position change



position change

96

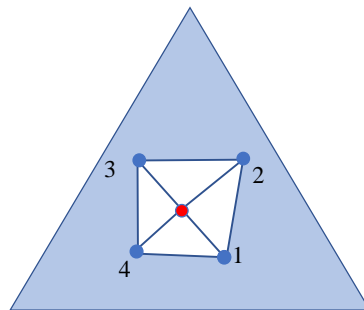
A closer look at the position change



position change

97

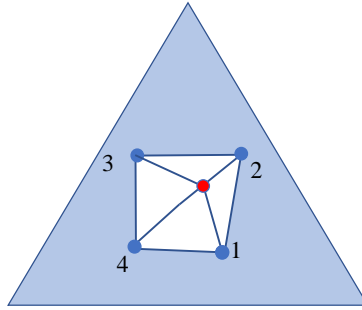
A closer look at the position change



position change

98

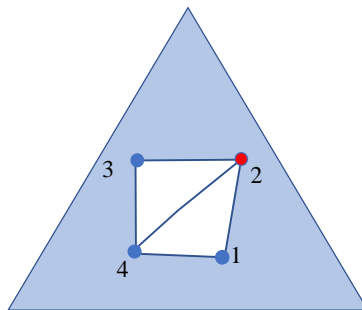
A closer look at the position change



position change

99

A closer look at the position change



position change

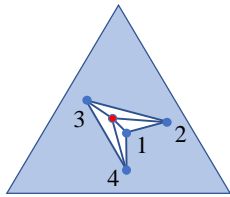
100

Cairn's double recursion approach

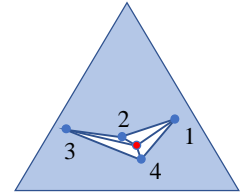
number $S(n)$ of morphing steps

$$S(n) = 2S(n - 1) + O(1)$$

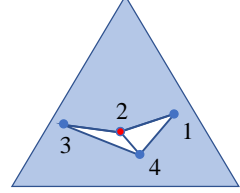
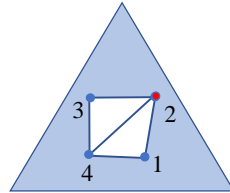
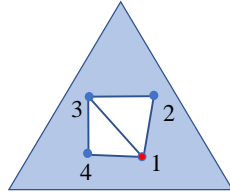
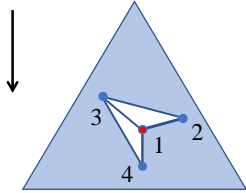
$$S(n) \in O(2^n)$$



contraction $O(1)$



expansion $O(1)$



recursion
 $\xrightarrow{S(n-1)}$

position change
 $\xrightarrow{O(1)}$

recursion
 $\xrightarrow{S(n-1)}$

Straight-line drawings of a triangulation G with n vertices in R^{2n}

each point is a straight-line drawing of G

planar drawings with outer face f_1

planar drawings with outer face f_2

planar drawings with outer face f_3

planar drawings with outer face f_4

connected - Cairns 1944
unbounded - scale up

Hooper Gotsman 1999 curve

- Tutte 1963 Drawing
- Schnyder 1990 Drawing
- de Frasseix Pach Pollack 1990 Drawing

Floater Gotsman Curve – outer face

Suppose the outer face has the same drawing in Γ_0 and in Γ_1

103

Floater Gotsman Curve – drawing Γ_0

Determine coefficients $\lambda_{ij}(0)$ such that Γ_0 is a weighted barycentric drawing with weights $\lambda_{ij}(0)$

- Let x_i^0 be the coordinates of vertex i in Γ_0
- Compute positive values $\lambda_{ij}(0)$ so that

$$\sum_{j \in N(i)} \lambda_{ij}(0) x_j^0 = x_i^0 \quad \text{and} \quad \sum_{j \in N(i)} \lambda_{ij}(0) = 1$$

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Floater Gotsman Curve – drawing Γ_1

Determine coefficients $\lambda_{ij}(1)$ such that Γ_1 is a weighted barycentric drawing with weights $\lambda_{ij}(1)$

- Let x_i^1 be the coordinates of vertex i in Γ_1
- Compute positive values $\lambda_{ij}(1)$ so that

$$\sum_{j \in N(i)} \lambda_{ij}(1) x_j^1 = x_i^1 \quad \text{and} \quad \sum_{j \in N(i)} \lambda_{ij}(1) = 1$$

105

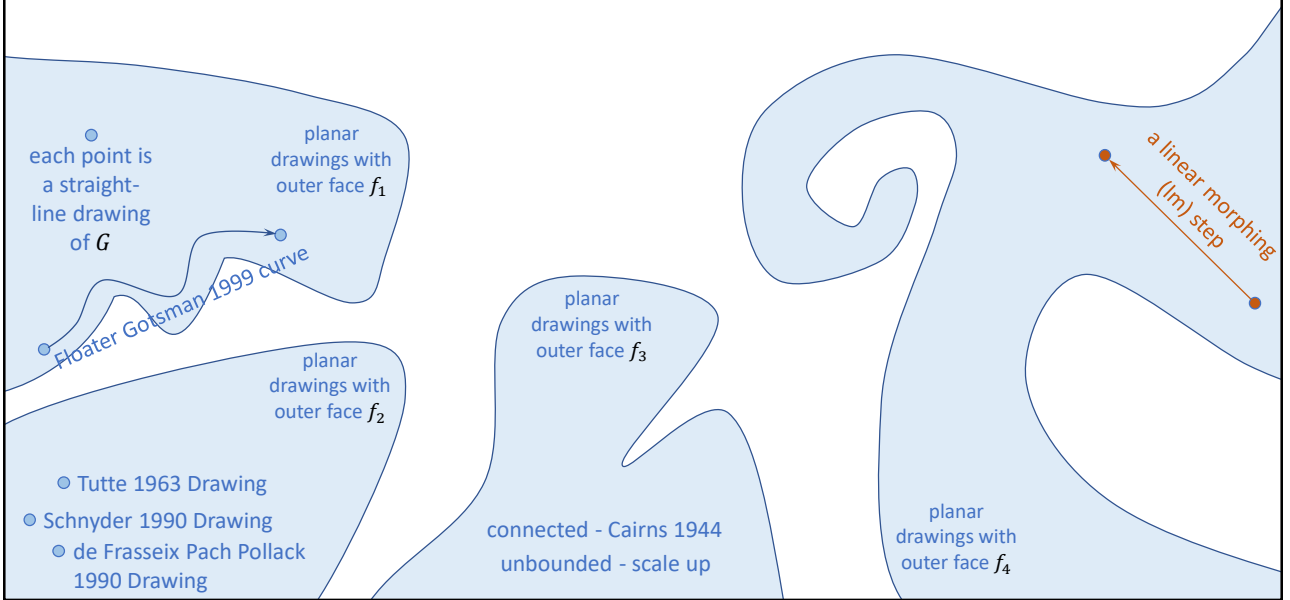
Floater Gotsman Curve

- At each time t , Γ_t is a weighted barycentric (planar) drawing with weights

$$\lambda_{ij}(t) = \lambda_{ij}(0)(1 - t) + \lambda_{ij}(1)t$$

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Straight-line drawings of a triangulation G with n vertices in R^{2n}



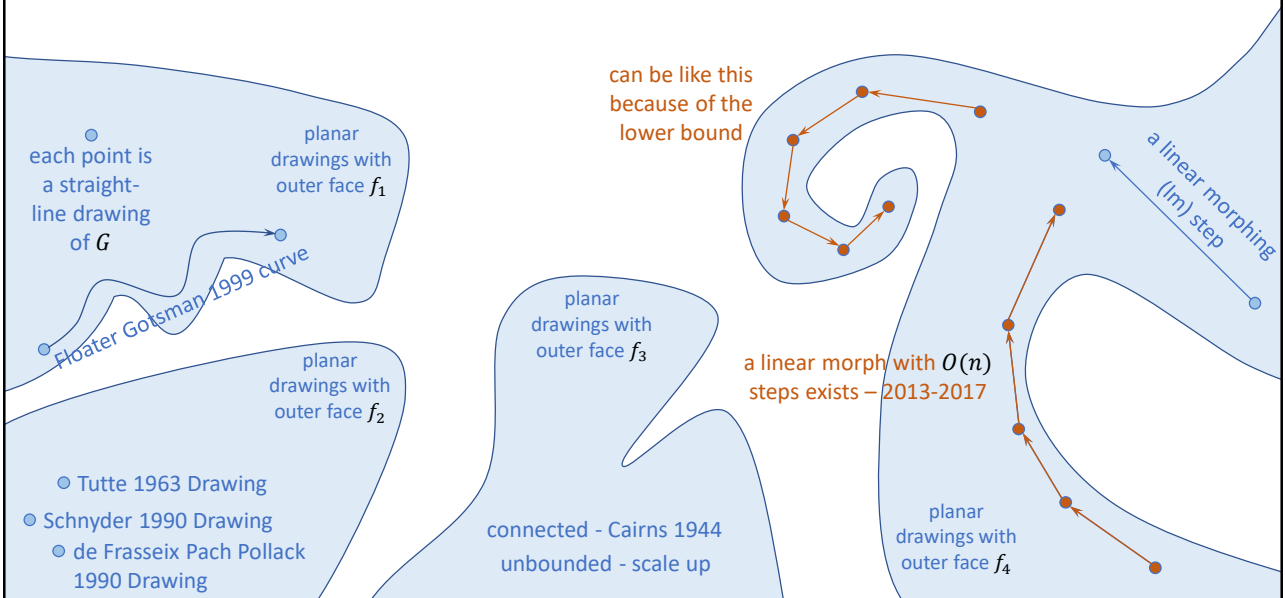
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Linear morphs and morphing steps

- In a *linear morphing step* every vertex moves along a straight-line segment at uniform speed
 - Vertices may move at different speeds, and some vertices may remain stationary
- A *linear morph* consists only of linear morphing steps

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Straight-line drawings of a triangulation G with n vertices in R^{2n}



109

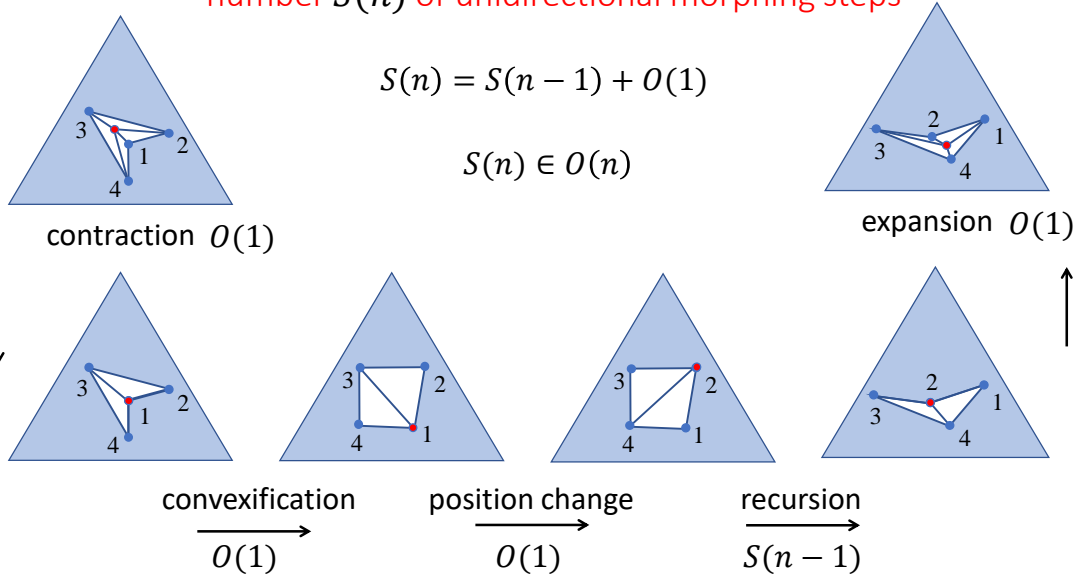
a morph with $O(n)$ lm steps exists - 2013-2017

- S. Alamdari, P. Angelini, T.M. Chan, gdb, F. Frati, A. Lubiw, M. Patrignani, V. Roselli, S. Singla, and B. T. Wilkinson, Morphing planar graph drawings with a polynomial number of steps, SODA 2013
- P. Angelini, F. Frati, M. Patrignani, and V. Roselli, Morphing planar graph drawings Efficiently, GD 2013
- P. Angelini, G. Da Lozzo, gdb, F. Frati, M. Patrignani, and V. Roselli, Morphing planar graph drawings optimally, ICALP 2014
- S. Alamdari, P. Angelini, F. Barrera-Cruz, T.M. Chan, G. Da Lozzo, gdb, F. Frati, P. Haxell, A. Lubiw, M. Patrignani, V. Roselli, S. Singla, B.T. Wilkinson. How to morph planar graph drawings, SICOMP 2017

110

Morphing triangulations with a few steps

number $S(n)$ of unidirectional morphing steps



111

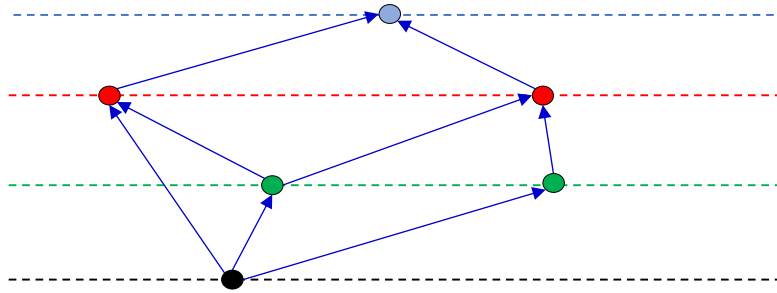
A convexification tool

- Given:
 - a triconnected plane graph $G = (V, E)$
 - a set L of parallel lines
 - a mapping of the vertices of V to lines of L such that orienting the edges of E according to the order of the lines in L yields an st -orientation of G
- Then G admits a convex drawing (all faces are convex polygons) in which each vertex of V lies on the line of L it is mapped to

Hong Nagamochi, Convex drawings of hierarchical planar graphs and clustered planar graphs, JDA 2010

112

A convexification tool



This slide: Courtesy of Patrizio Angelini

113

Straight-line drawings of a triangulation G with n vertices in R^{2n} problems

each point is a straight-line drawing of G

planar drawings with outer face f_1

planar drawings with outer face f_2

planar drawings with outer face f_3

planar drawings with outer face f_4

connected - Cairns 1944

unbounded - scale up

can be like this because of the lower bound

a linear morphing (lm) step

a morph with $O(n)$ lm steps exists - Alamdari et al. 2017

Fleater Gotsman 1999 curve

Is it possible to make it piecewise linear?

- Tutte 1963 Drawing
- Schnyder 1990 Drawing
- de Frasseix Pach Pollack 1990 Drawing

114

Straight-line drawings of a triangulation G with n vertices in R^{2n} problems

each point is a straight-line drawing of G

planar drawings with outer face f_1

planar drawings with outer face f_2

planar drawings with outer face f_3

planar drawings with outer face f_4

region of the non-planar drawings - connected?

connected - Cairns 1944
unbounded - scale up

can be like this because of the lower bound

a morph with $O(n)$ l_m steps exists - Alamdari et al. 2017

a linear morphing (l_m) step

does this exist?

Hofer Gotsman 1999 curve

- Tutte 1963 Drawing
- Schnyder 1990 Drawing
- de Frasseix Pach Pollack 1990 Drawing

115

Straight-line drawings of a triangulation G with n vertices in R^{2n} problems

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a morph with $O(n)$ l_m steps exists - Alamdari et al. 2017

a linear morphing (l_m) step

does this exist?

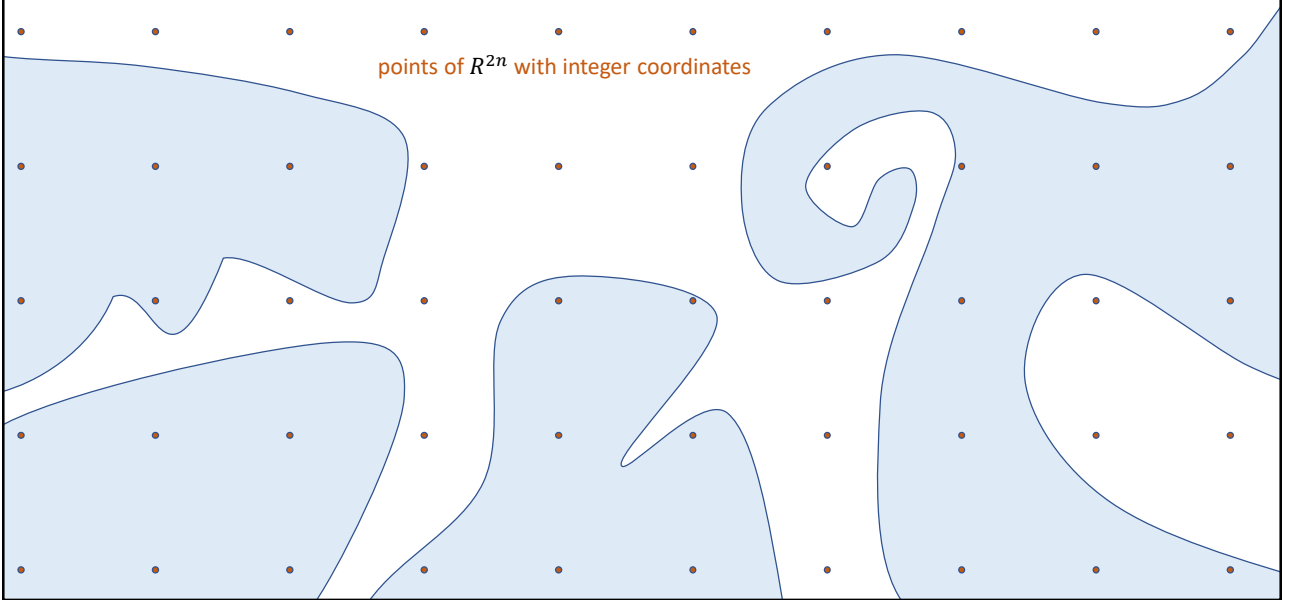
Hofer Gotsman 1999 curve

- Tutte 1963 Drawing
- Schnyder 1990 Drawing
- de Frasseix Pach Pollack 1990 Drawing

how to traverse this? Angelini Cortese gdb Patrignani, Topological Morphing of Planar Graphs, TCS 2013

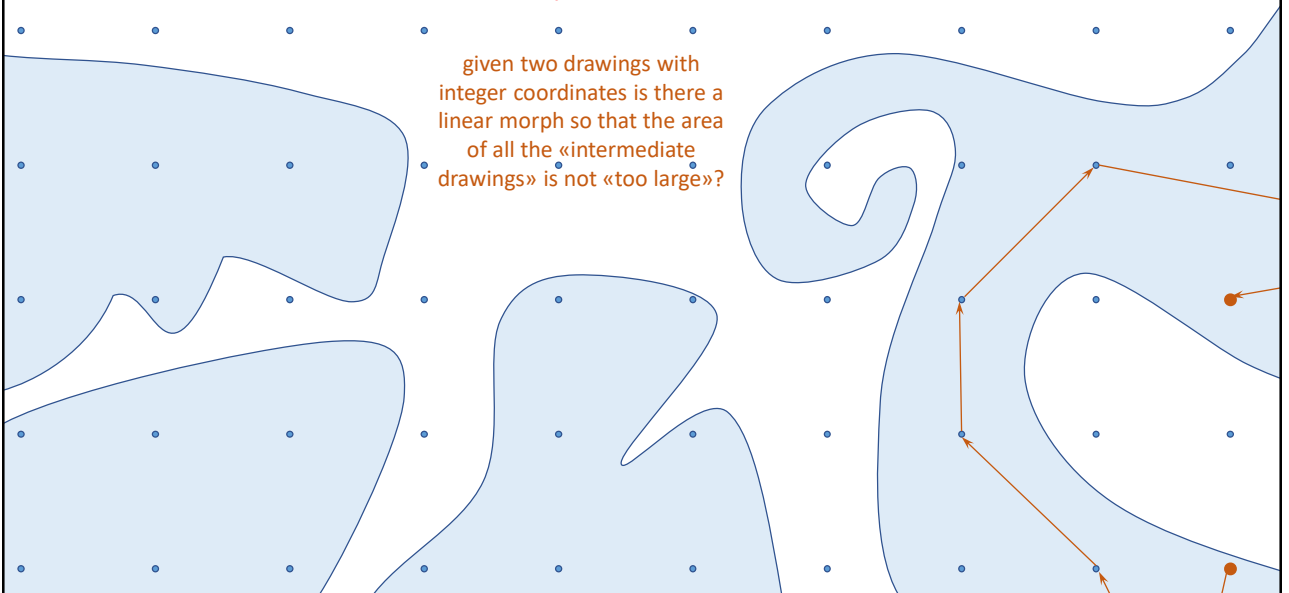
116

Straight-line drawings of a triangulation G with n vertices in R^{2n}



117

Straight-line drawings of a triangulation G with n vertices in R^{2n} problems



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morphs

upward planar straight-line drawings
of upward-planar graphs

119

Upward straight-line drawings of an upward planar graph G with
 n vertices in R^{2n}

•
each point is
an upward
planar
straight-line
drawing of G

Upward
equivalent
planar straight-
line drawings

Upward
equivalent
planar straight-
line drawings

Upward
equivalent
planar straight-
line drawings

Upward
equivalent
planar straight-
line drawings

120

Upward straight-line drawings of an upward planar graph G with n vertices in R^{2n}

each point is an upward planar straight-line drawing of G

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

connected – da lozzo gdb frati patrigani roselli 2018

unbounded - scale up

can be like this because of the lower bound

121

Upward straight-line drawings of an upward planar graph G with n vertices in R^{2n}

each point is an upward planar straight-line drawing of G

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

Upward equivalent planar straight-line drawings

connected – da lozzo gdb frati patrigani roselli 2018

unbounded - scale up

can be like this because of the lower bound

a morph with $O(n^2)$ lm steps exists – da lozzo gdb frati patrigani roselli 2018

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morphs

upward planar straight-line drawings of rooted trees

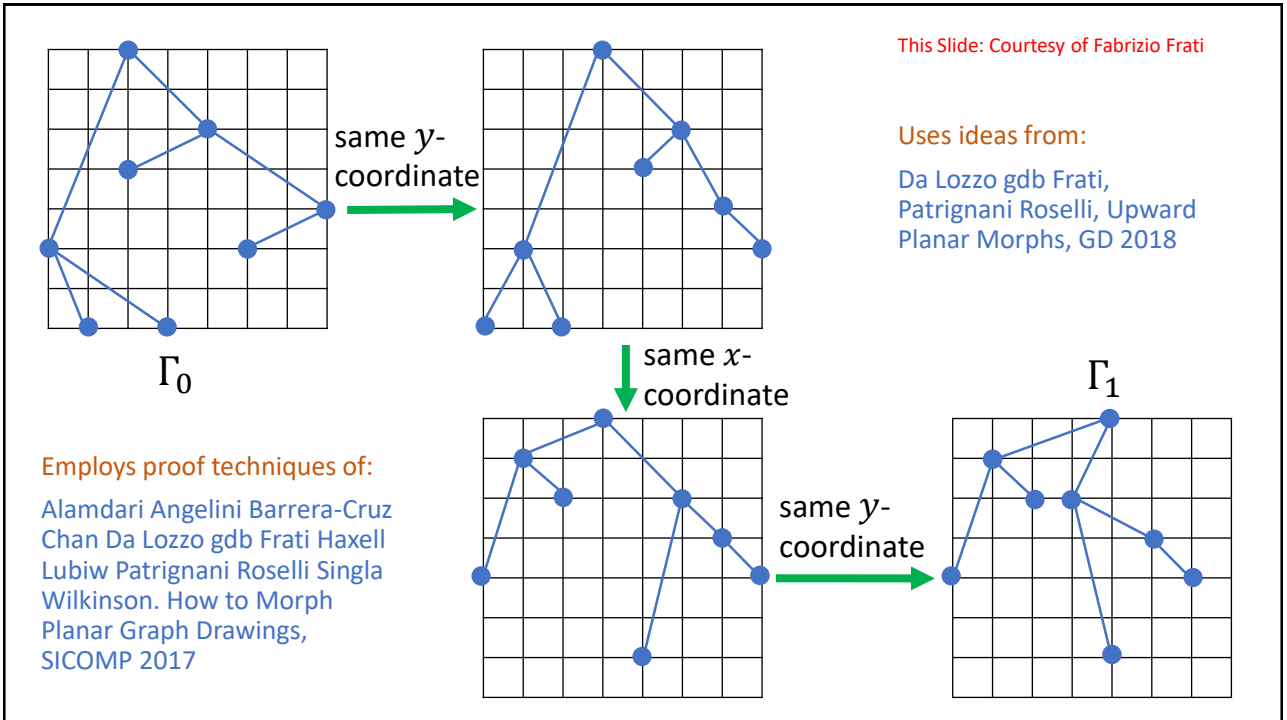
123

Upward straight-line drawings of a rooted binary tree T with n vertices in \mathbb{R}^{2n}

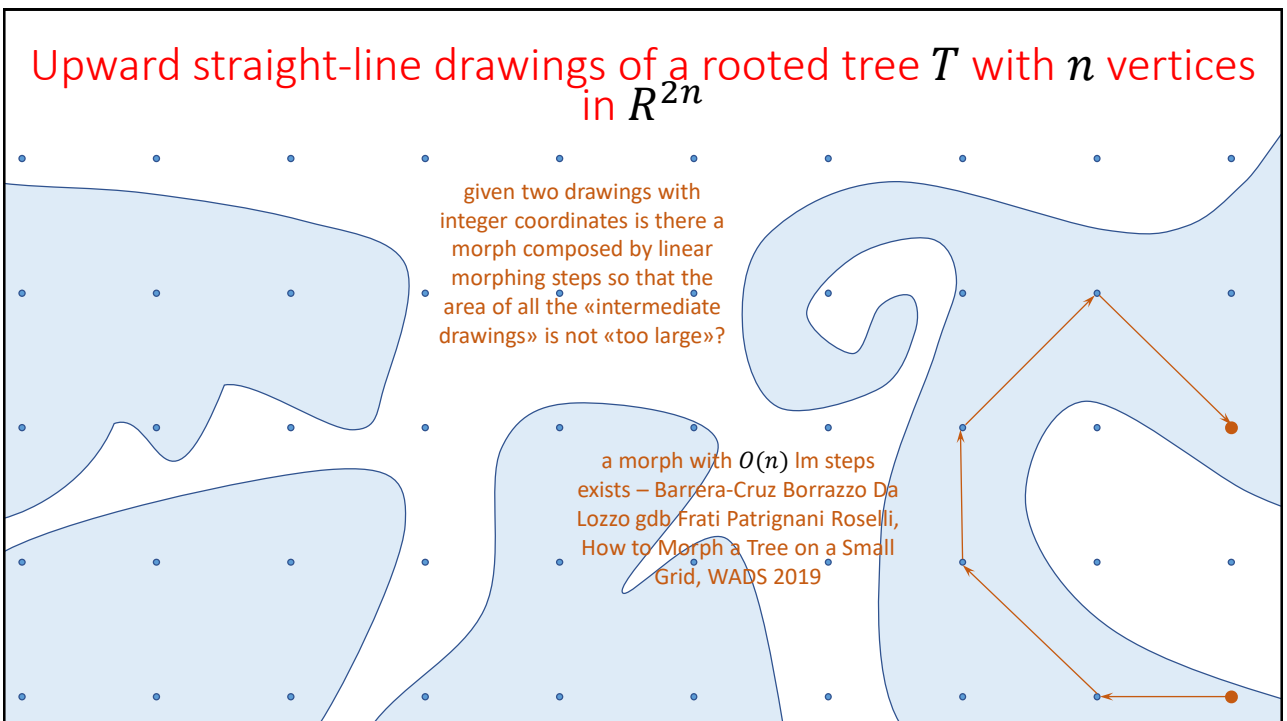
given two drawings with integer coordinates is there a linear morph so that the area of all the «intermediate drawings» is not «too large»?

a linear morph with 3 steps exists
Barrera-Cruz Borrazzo Da Lozzo
gdb Frati Patrignani Roselli, How
to Morph a Tree on a Small Grid,
WADS 2019

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125

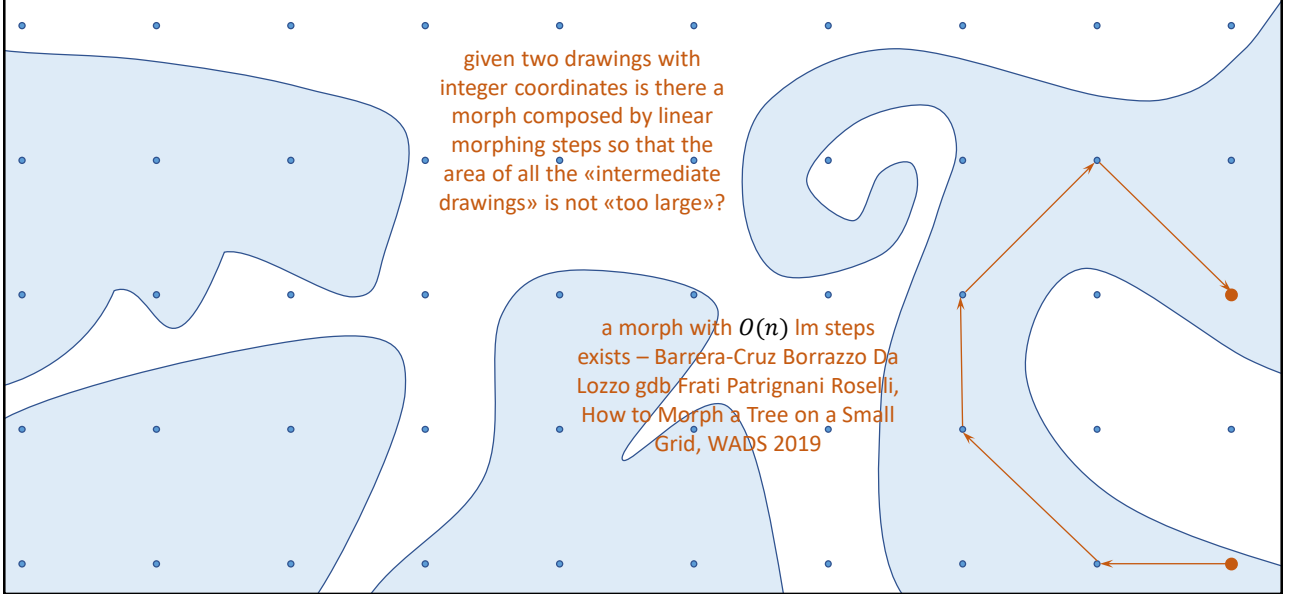


126

Straight-line drawings of a tree T with n vertices in R^{2n}

given two drawings with integer coordinates is there a morph composed by linear morphing steps so that the area of all the «intermediate drawings» is not «too large»?

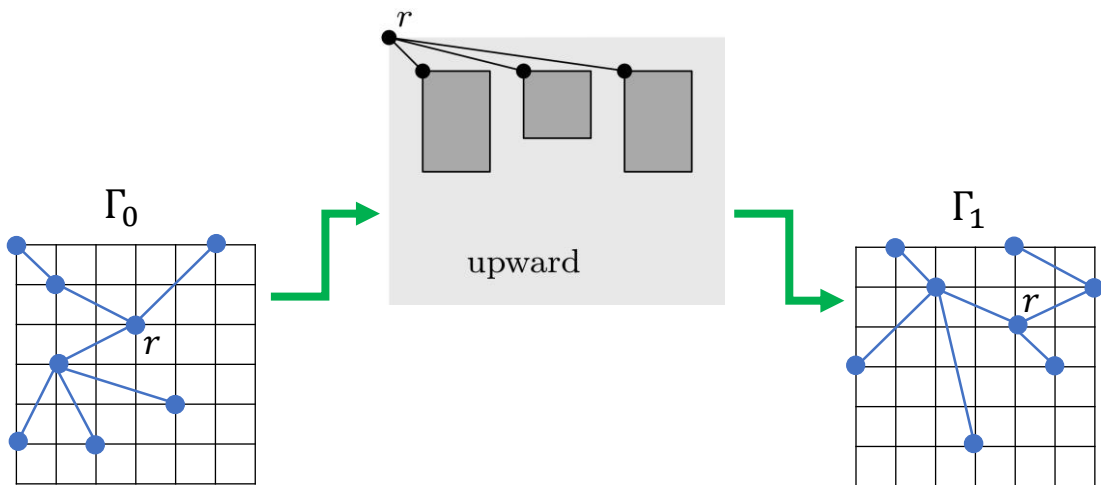
a morph with $O(n)$ lin steps exists – Barrera-Cruz Borrazzo Da Lozzo gdb Frati Patrignani Roselli, How to Morph a Tree on a Small Grid, WADS 2019



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General strategy

This Slide: Courtesy of Fabrizio Frati



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More GD topics that deal with time

- Simultaneous embedding with fixed edges (SEFE)
 - Time is in layers
- Phylogenetic Networks
 - Represent the evolution over time
- Greedy drawings
 - Time is in routing
- Constraints
 - Objects that do not change in the graph do not change in the drawing
- Drawing extensions
 - The graph grows during time

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More generally

- All Graph Drawing Papers consider time
- Some of them focus only on instant $t = 0$

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This Is Time for Graph Drawing

Giuseppe Di Battista
Università degli Studi Roma Tre

27th International Symposium on Graph Drawing and Network Visualization – GD2019
Průhonice/Prague, September 17-20, 2019