Bundled Crossings Revisited

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Motivation

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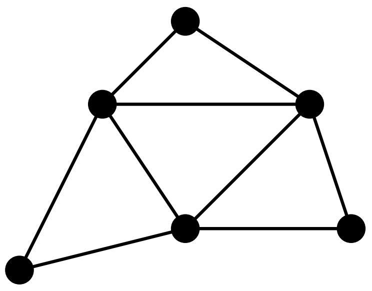

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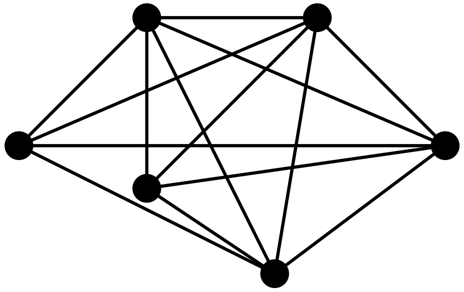

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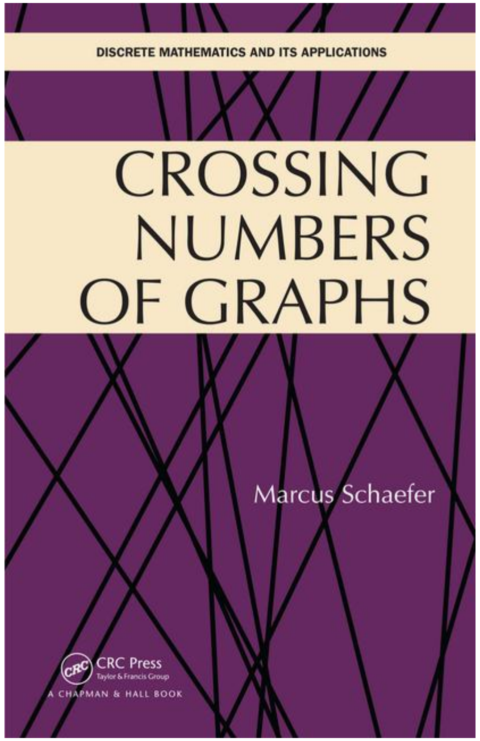

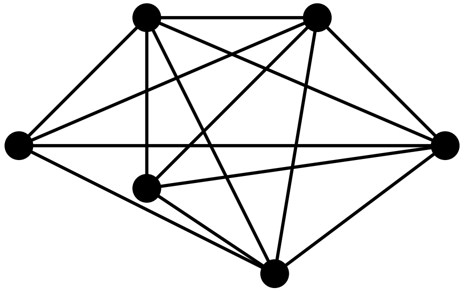

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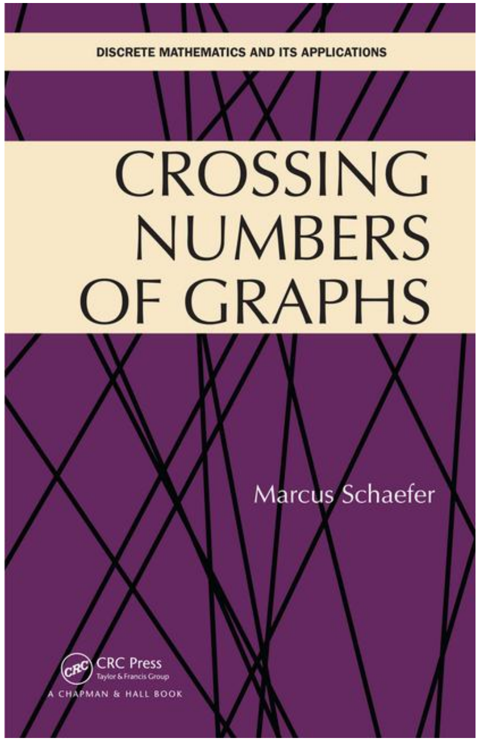

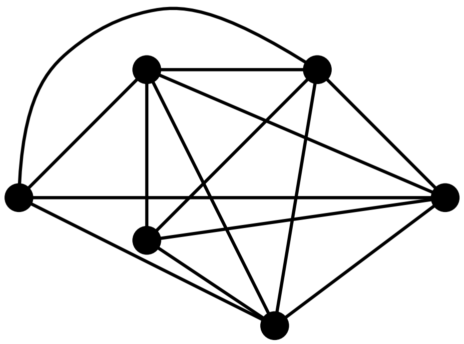

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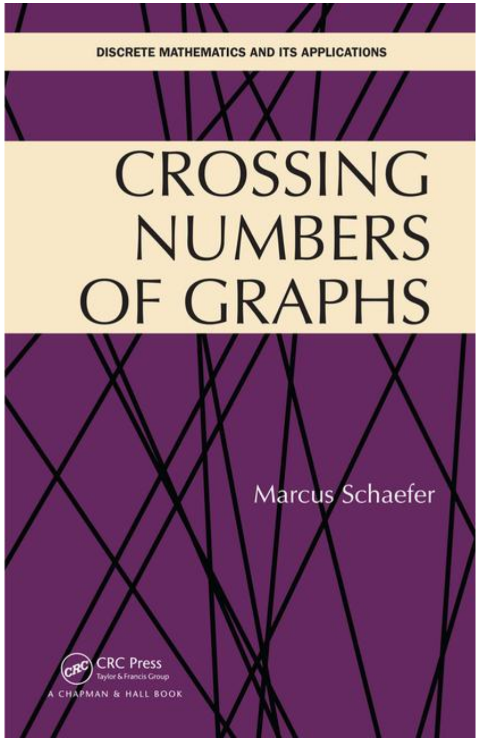

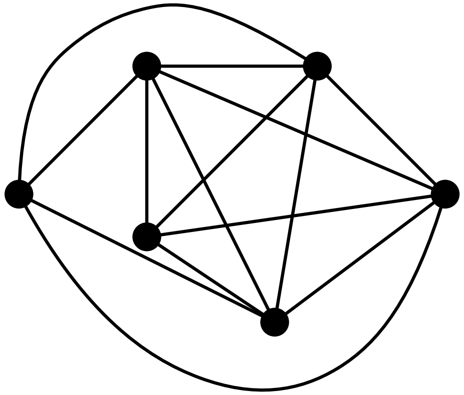

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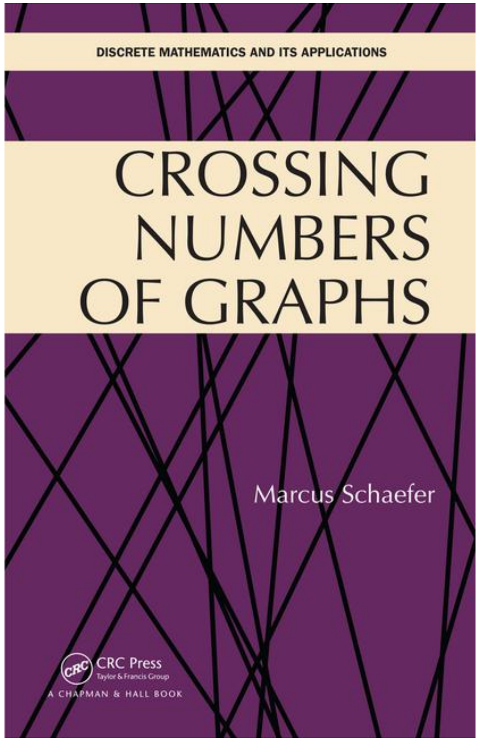

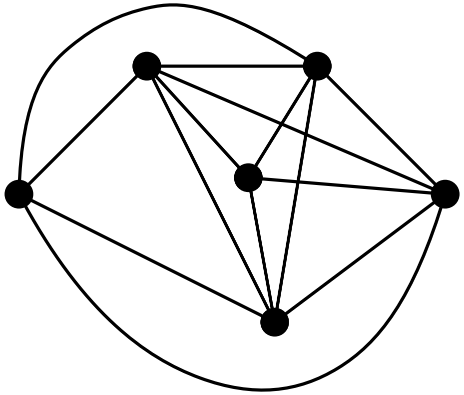

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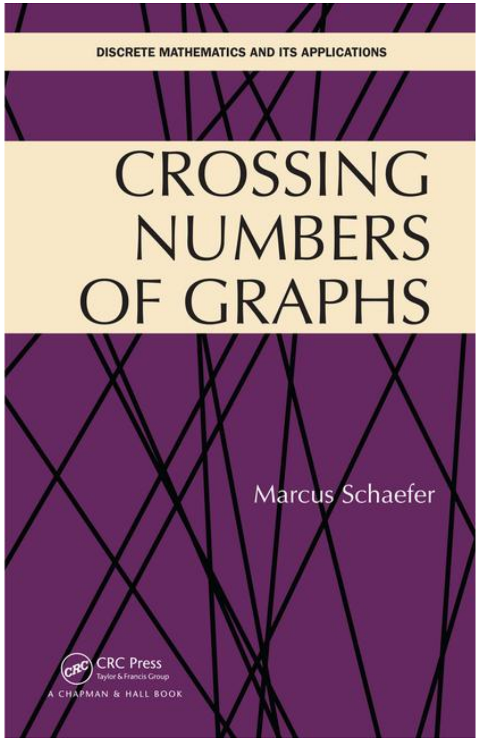

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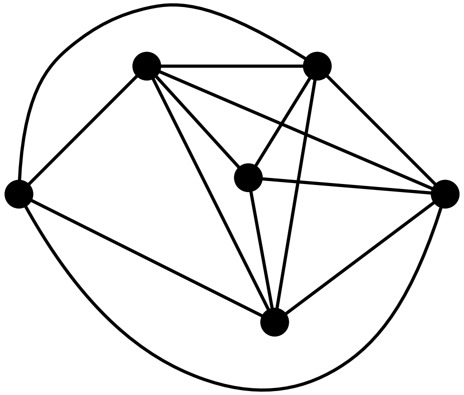

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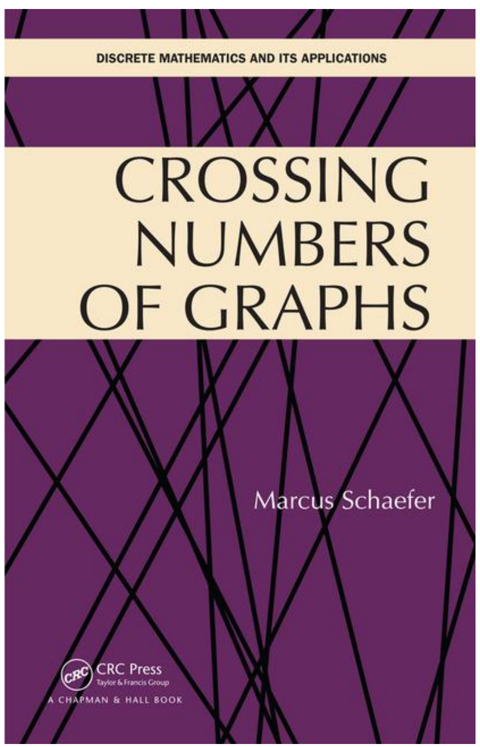
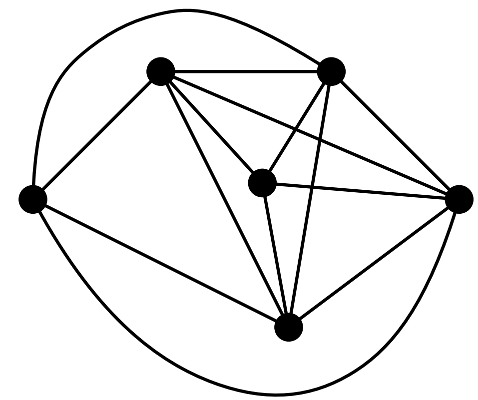
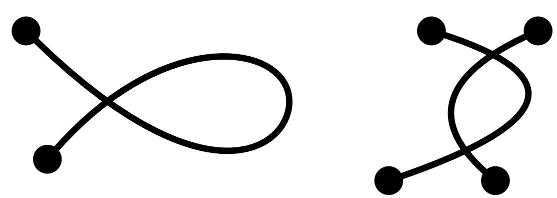


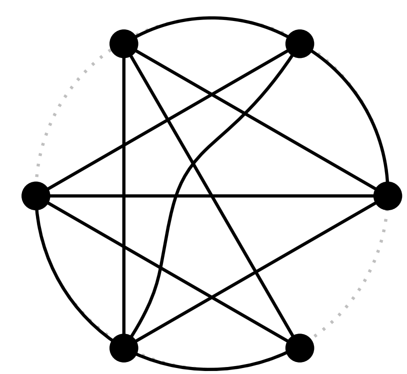


Lots of different variants.

Our main result concerns simple circular layouts.

simple avoids:





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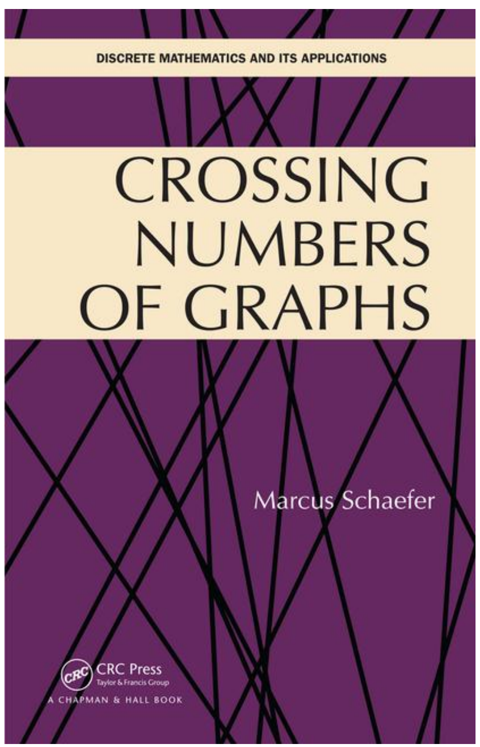

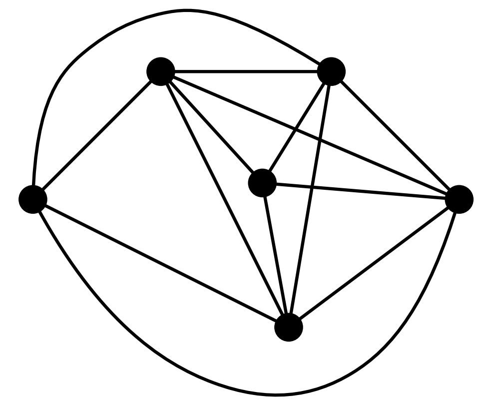

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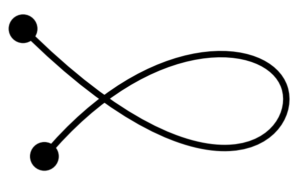
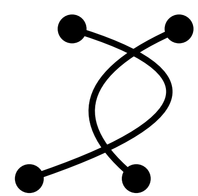
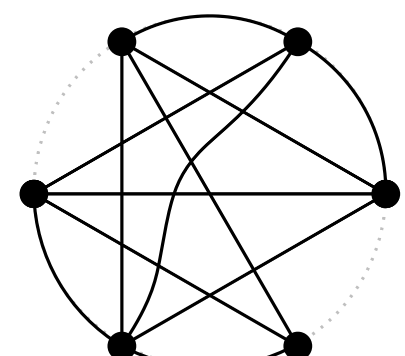
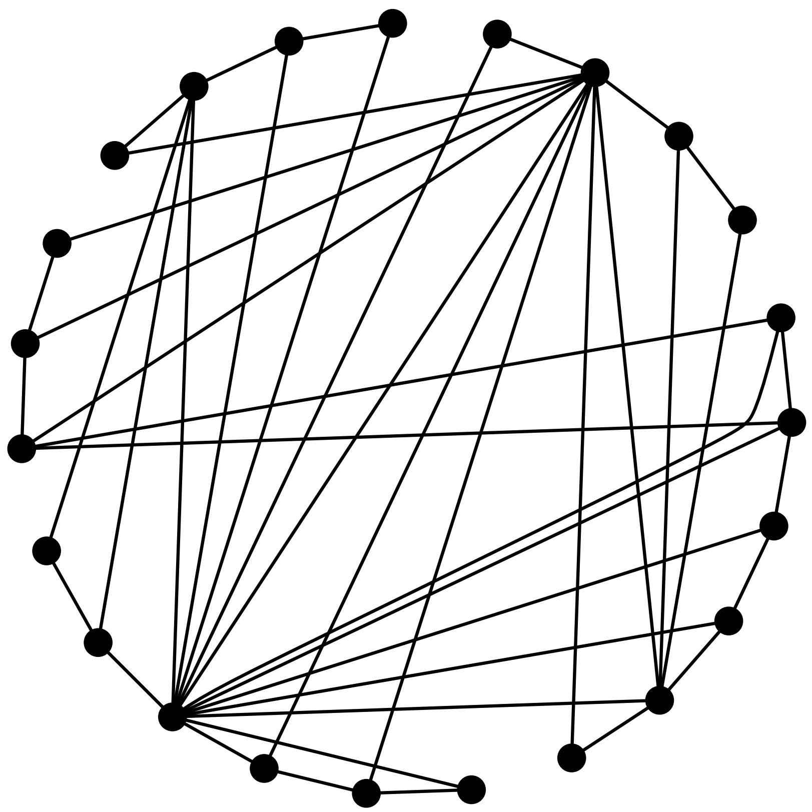
simple avoids:  



This talk concerns *bundled crossings*, def'd next.

Motivation



There is an FPT algorithm for deciding whether a graph admits a circular layout with k crossings.[Bannister, Eppstein '14]

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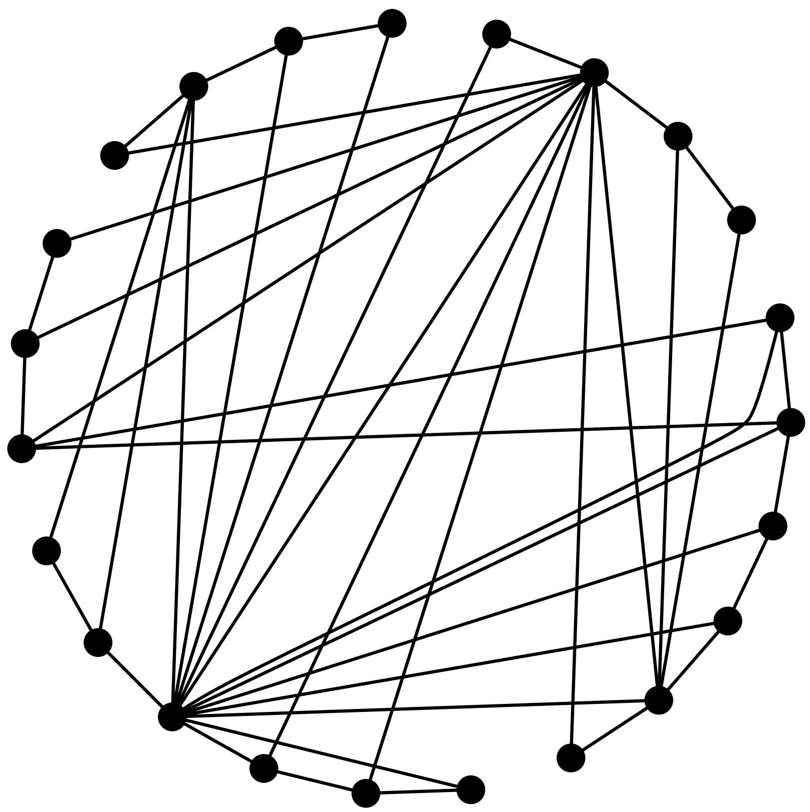
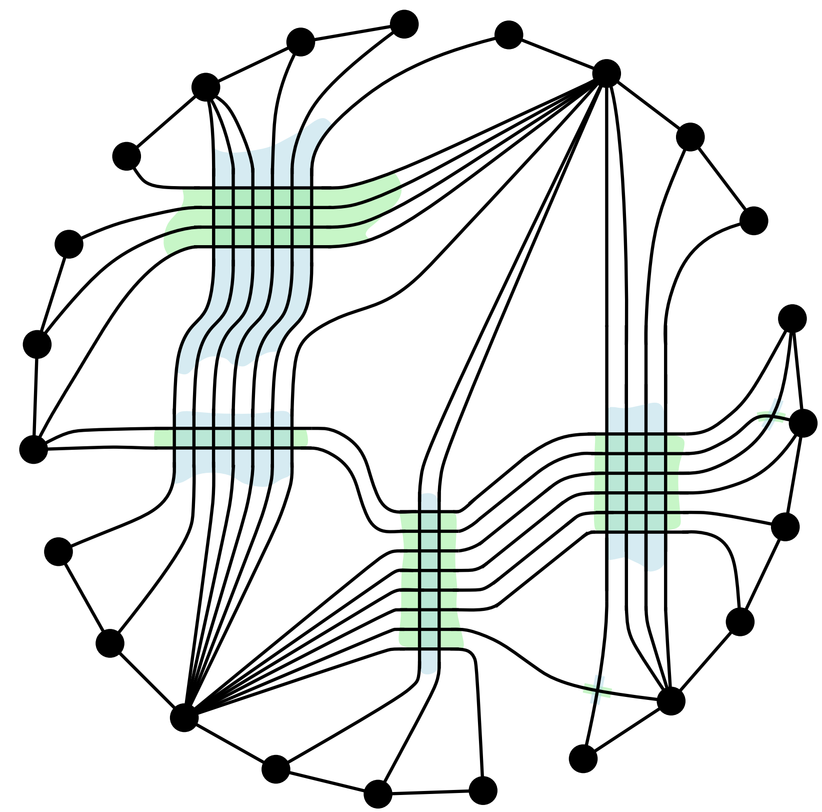


[Holten '06]

Bundle the drawing



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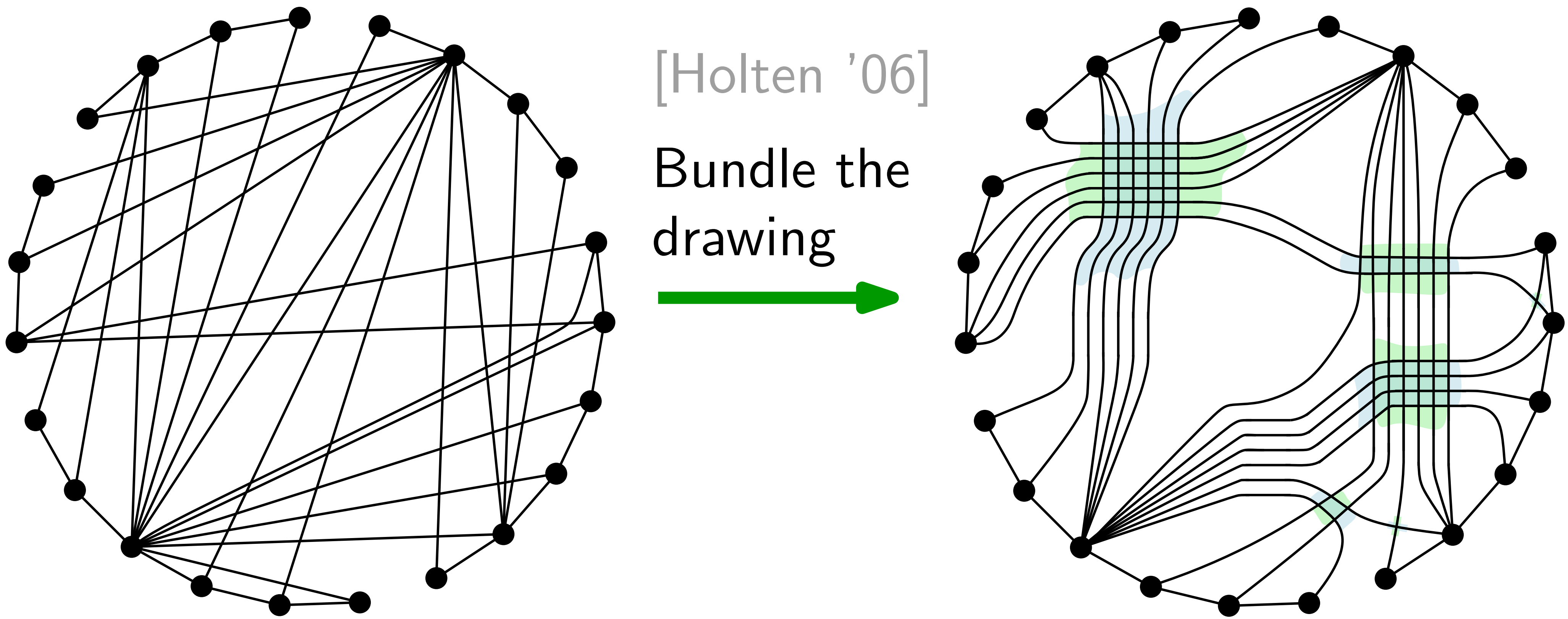

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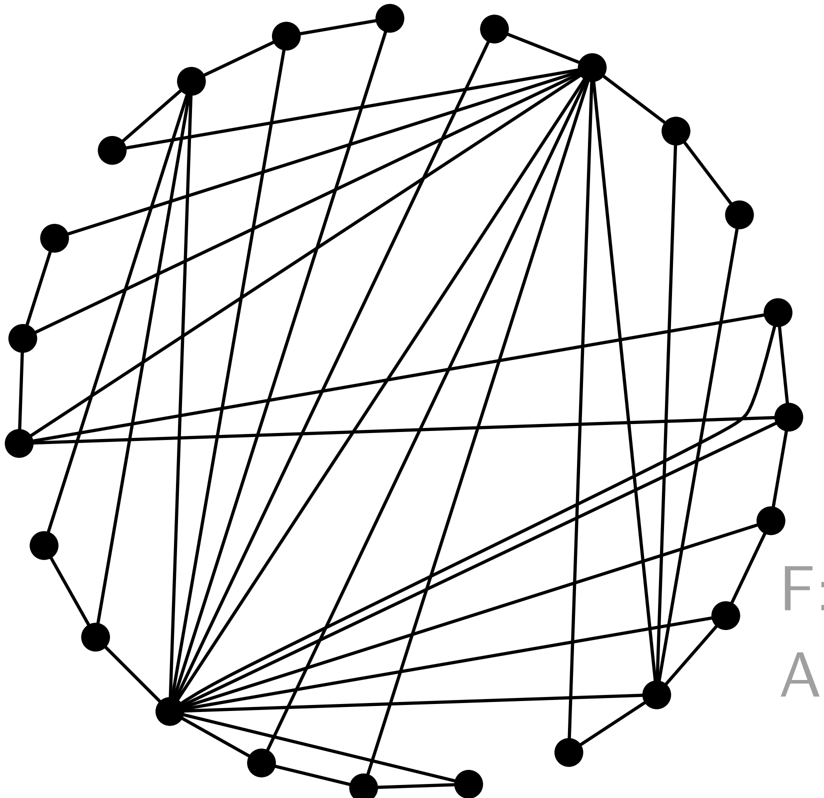
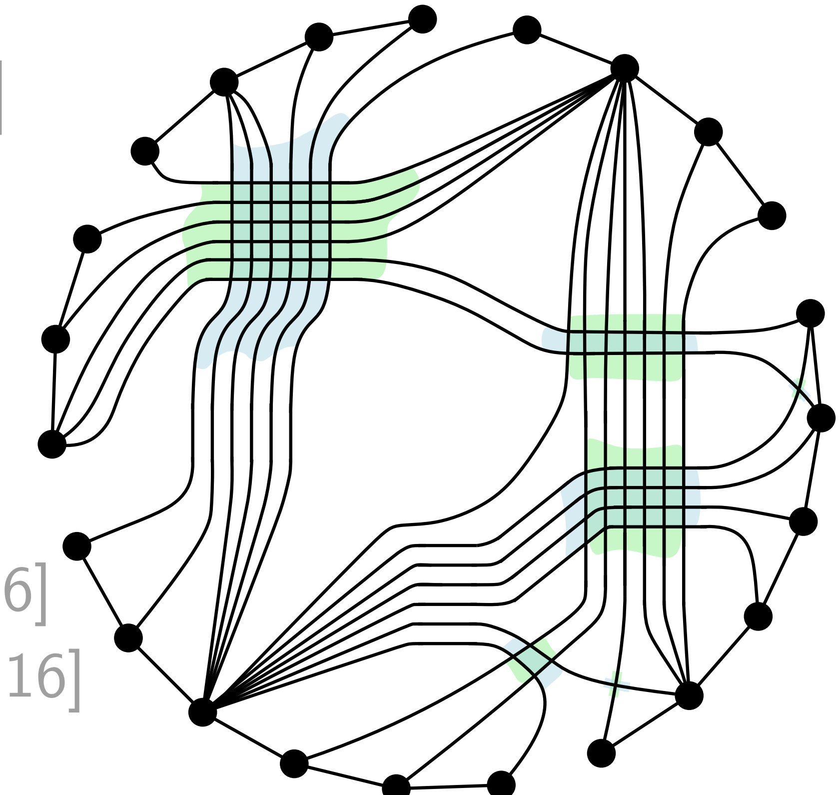


[Holten '06]

Bundle the drawing

F: [Fink et al. '16]

A: [Alam et al. '16]



Minimize crossings of bundles instead of edges!

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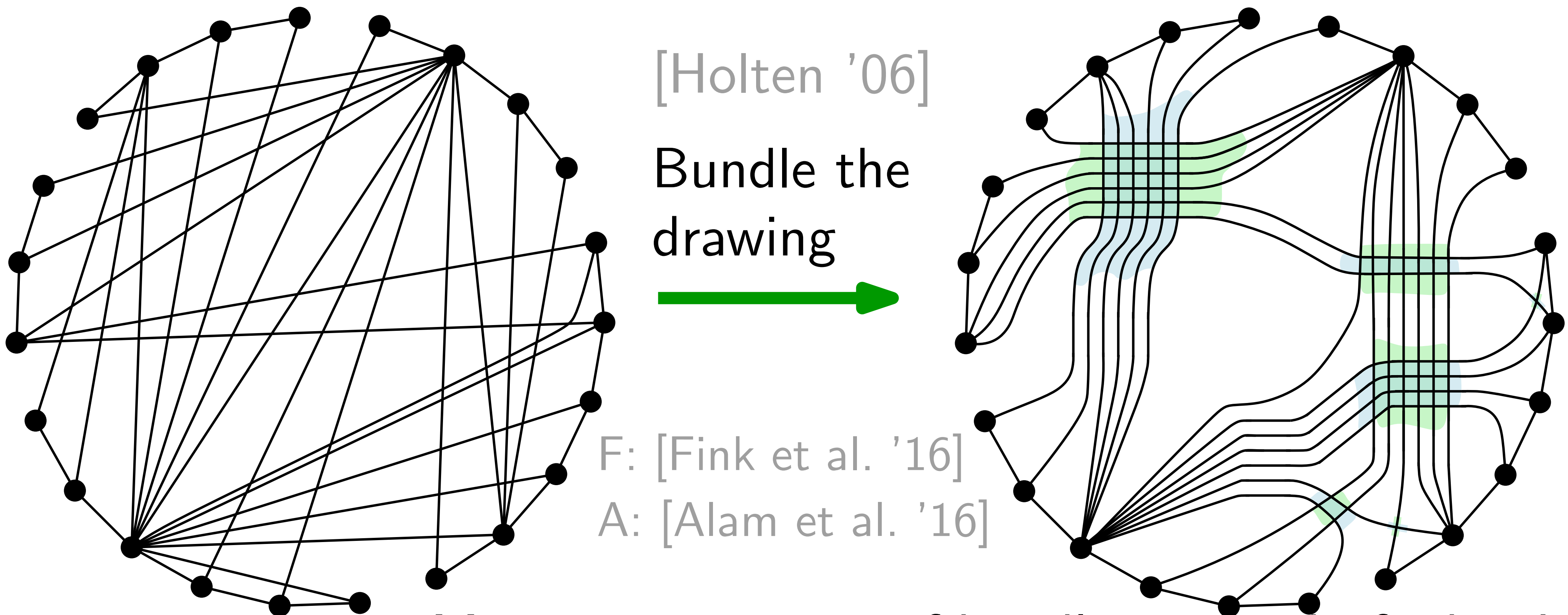

Minimize crossings of bundles instead of edges!

gen. layouts: NP-c for fixed [F] and variable [A] embeddings.

fixed embedding: 10-apx for circular, and $O(1)$-apx for gen. layouts [F]

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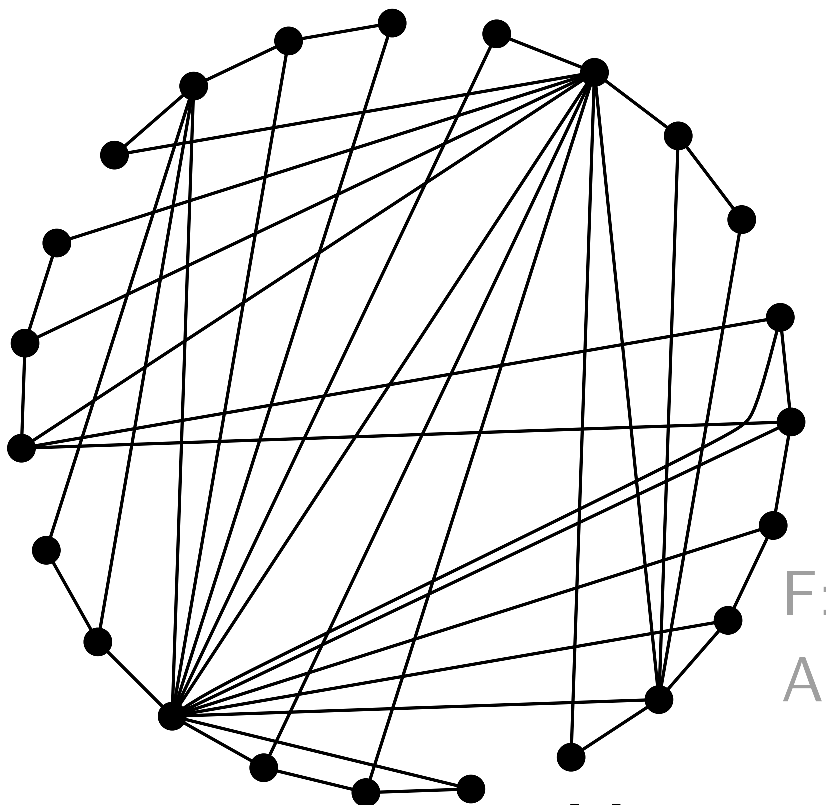

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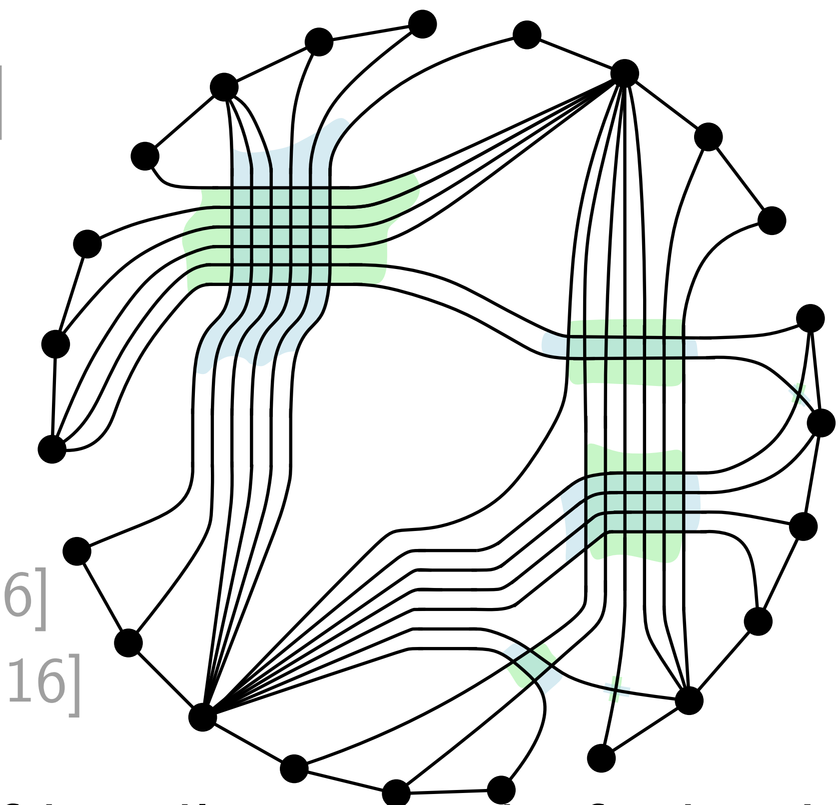

F: [Fink et al. '16]

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Minimize crossings of bundles instead of edges!

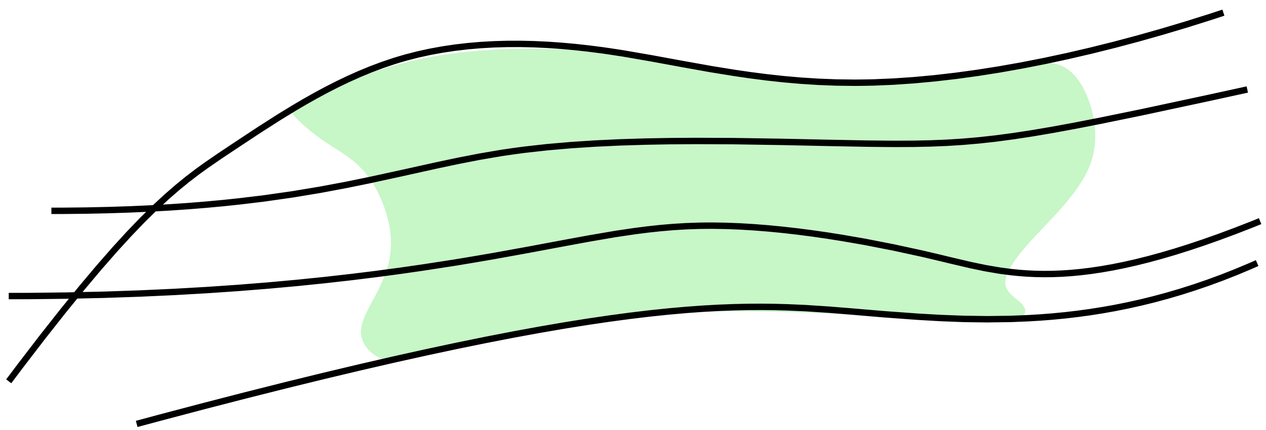
gen. layouts: NP-c for fixed [F] and variable [A] embeddings.

fixed embedding: 10-apx for circular, and $O(1)$-apx for gen. layouts [F]

Ques. Is there an FPT algorithm for deciding whether a graph admits a circular layout with k **bundled crossings**? [A]

Bundled Crossing

A **bundle** is a set of pieces of edges that travel in parallel in the drawing.



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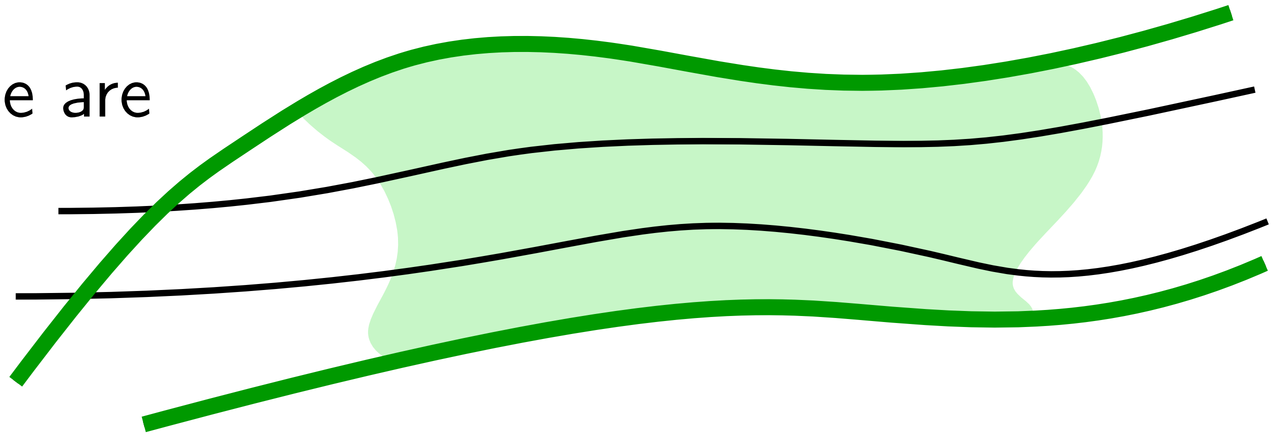
Outer edges of a bundle are called **frame edges**



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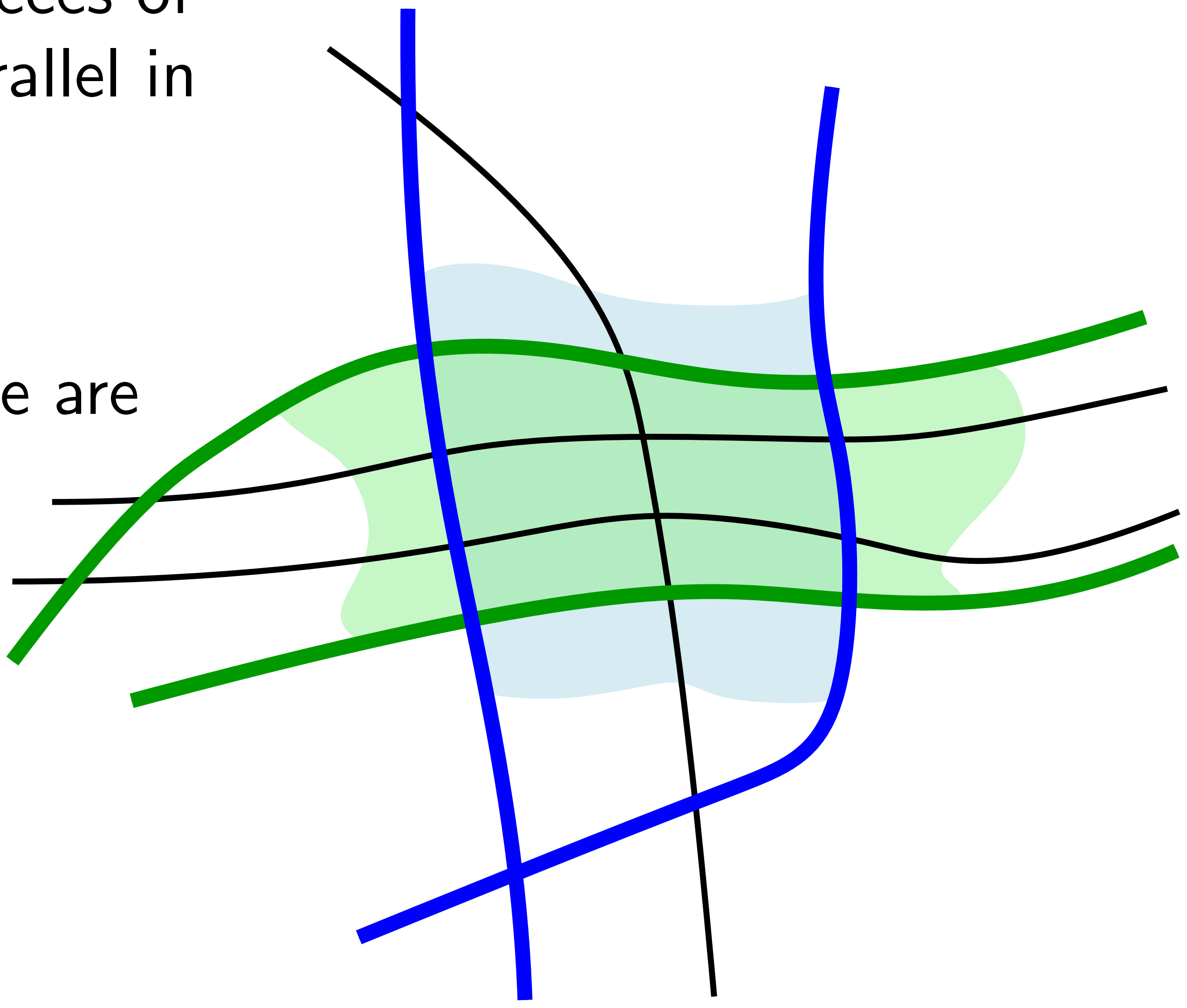

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A **bundled crosssing** is
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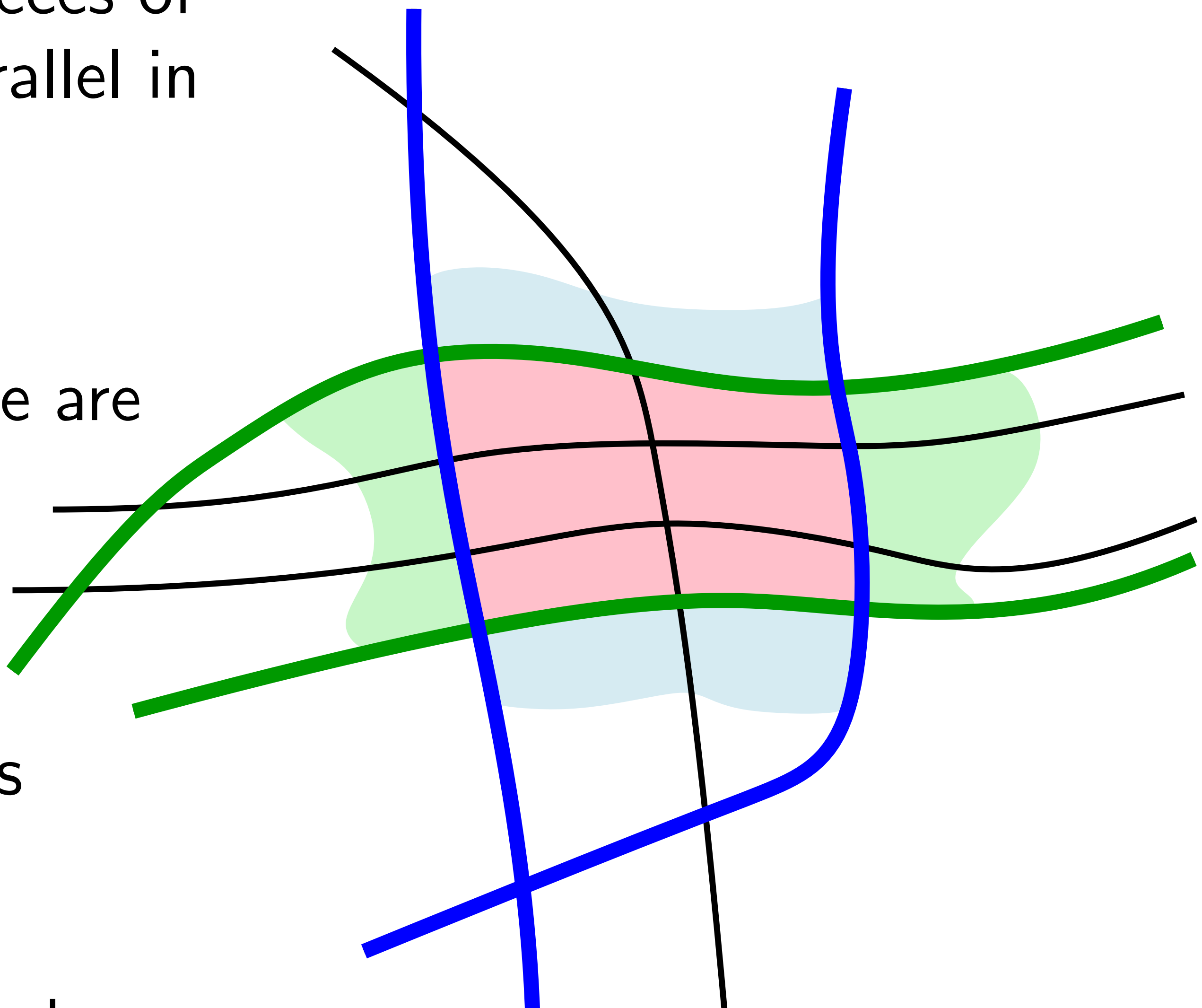
inside the region
bounded by the frame edges.



Bundled Crossing Minimization

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the minimum number of bundled crossings
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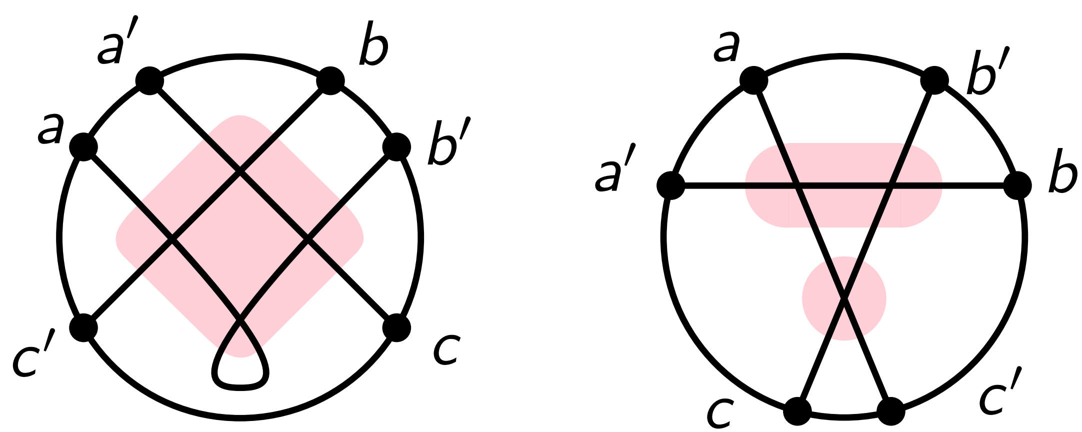
Remark on simple vs. non-simple: consider $K_{3,3}$



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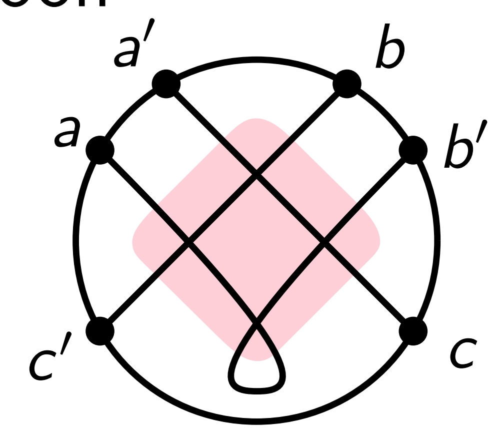
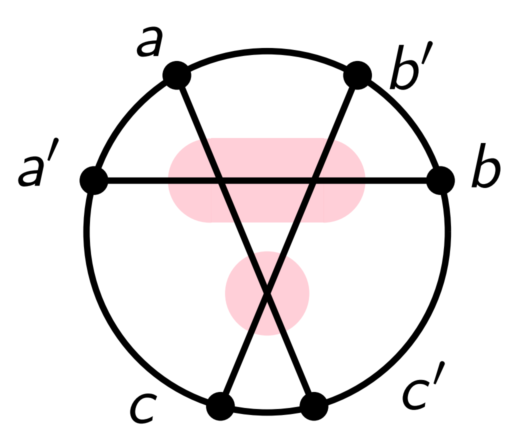
Remark on simple vs. non-simple: consider $K_{3,3}$

Non-simple $\rightsquigarrow$ orientable graph genus [Alam et al. 2016]

... more on this soon





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Other results (not covered in this talk, see the paper!):

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resolves open problem of [Fink et al.; 2016]

Thm. Deciding whether $\text{bc}^{\circ}(G) = k$ is FPT in k.

resolves an open problem of [Alam et al. 2016]

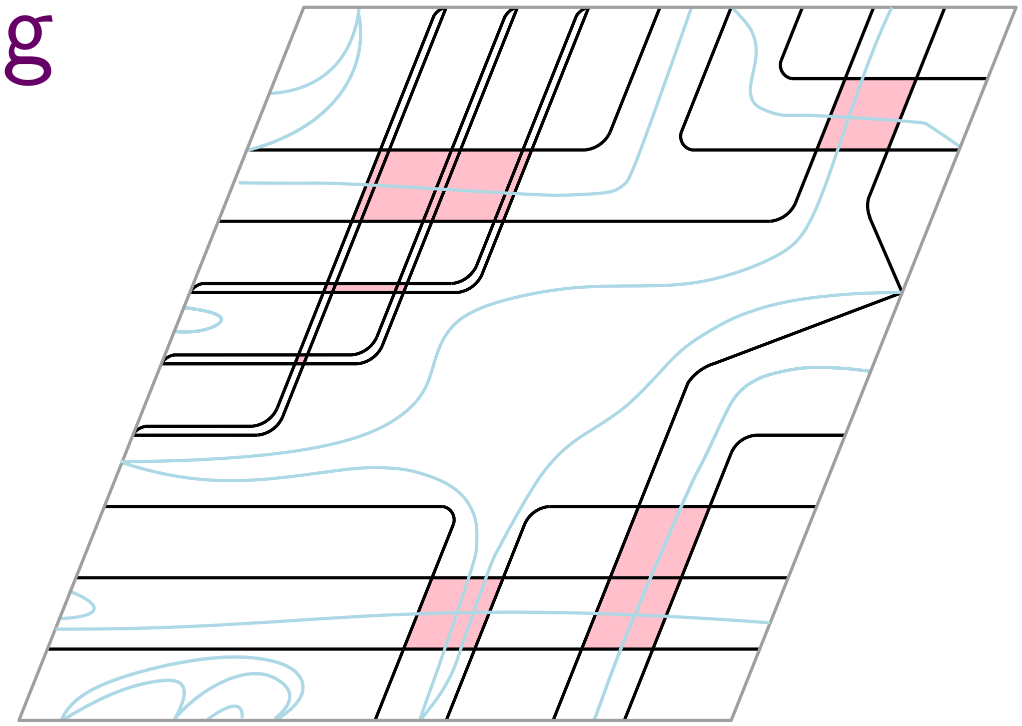
Other results (not covered in this talk, see the paper!):

Thm. For general layouts, on inputs (G, k), deciding whether G has a simple drawing with k bundled crossings is NPc. For non-simple, this is FPT in k (via genus).

Obs. For circular layouts, on inputs (G, k), deciding whether G has a (non-simple) circular drawing with k bundled crossings is FPT in k (via genus).

Structure of a drawing

Consider a drawing with k bundled crossings and observe that:



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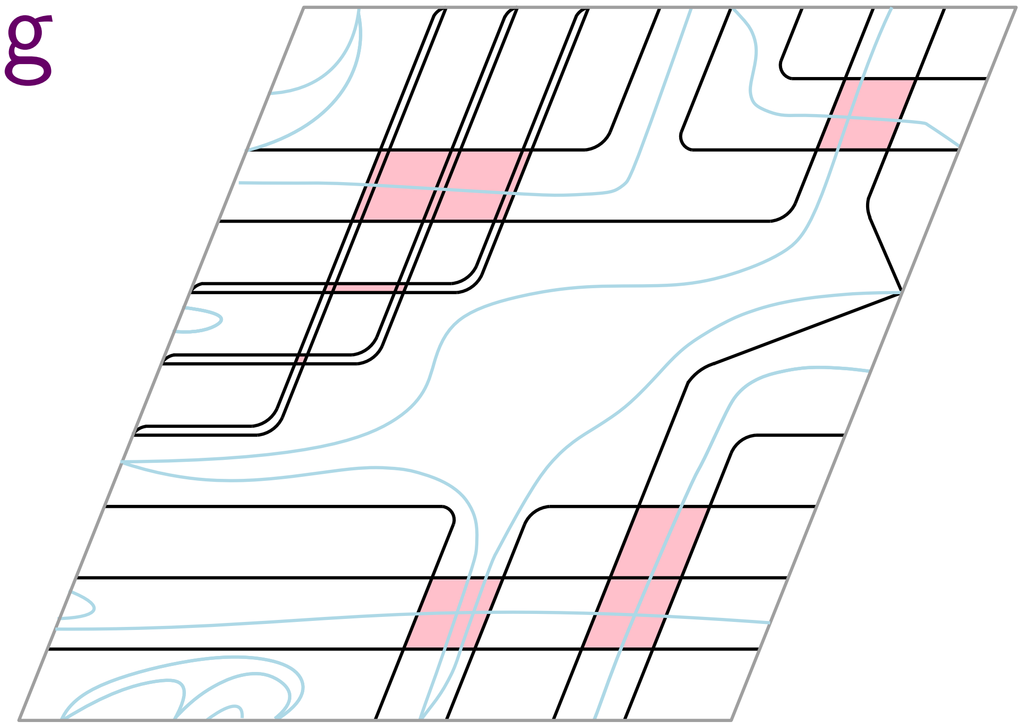

Consider a drawing
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and observe that:

- At most k bundled crossings $\implies$ at most $4k$ frame edges.

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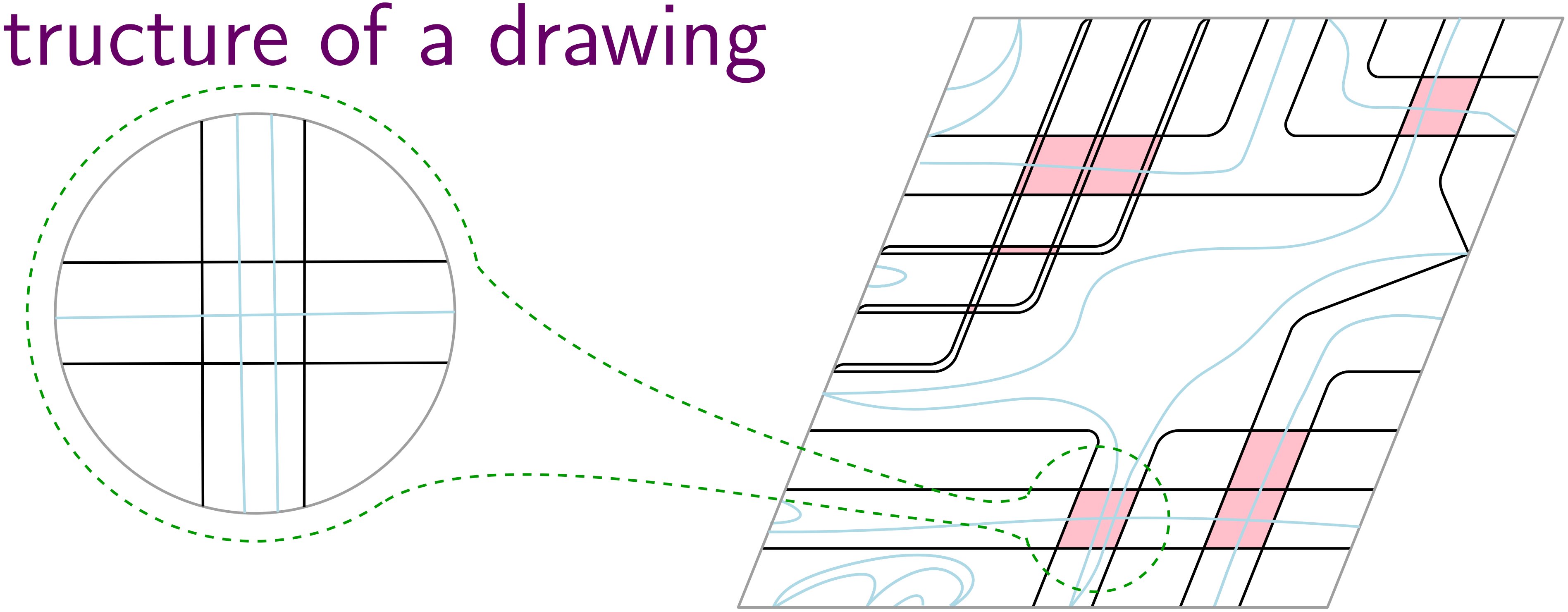

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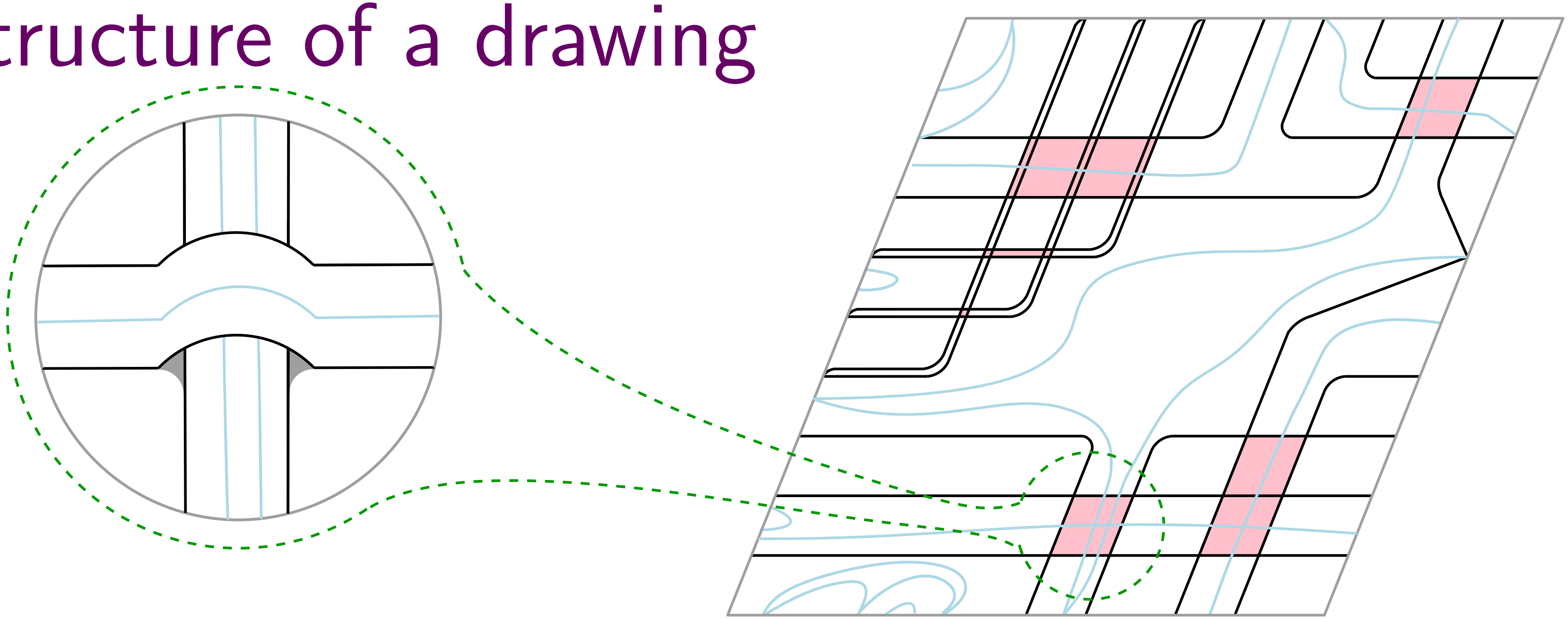

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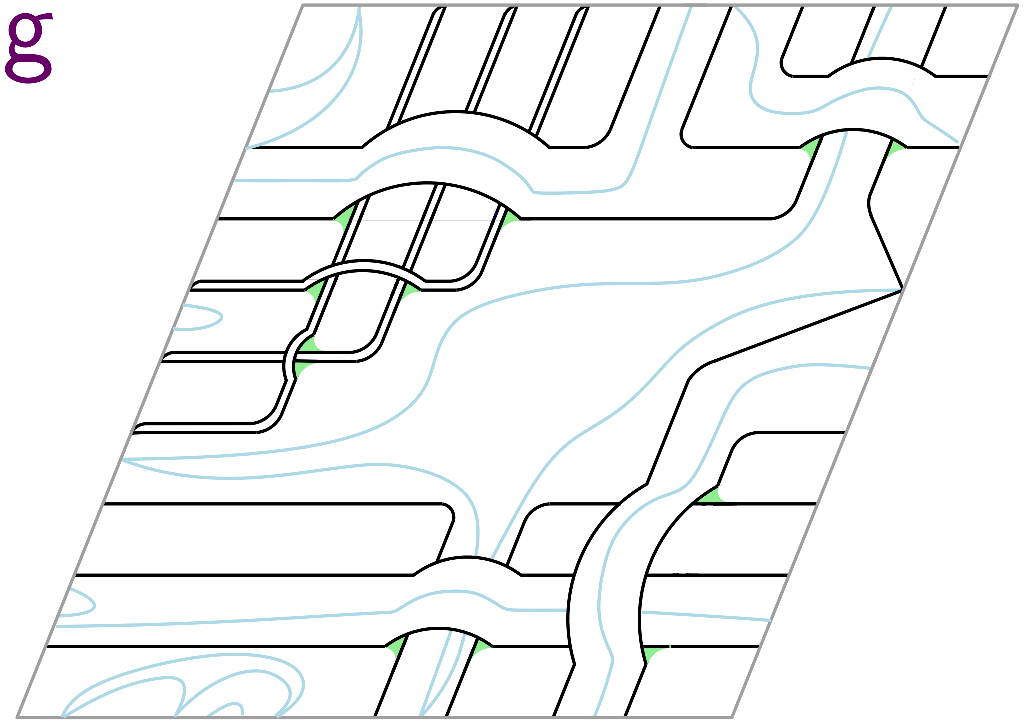

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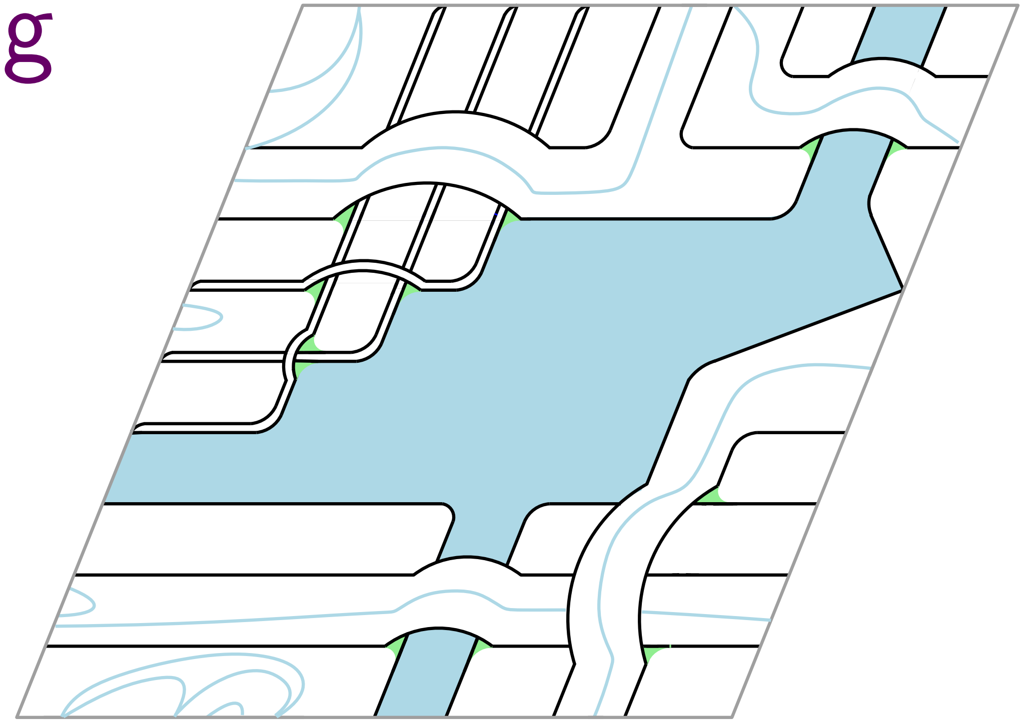

- At most k bundled crossings $\implies$ at most $4k$ frame edges.
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Structure of a drawing

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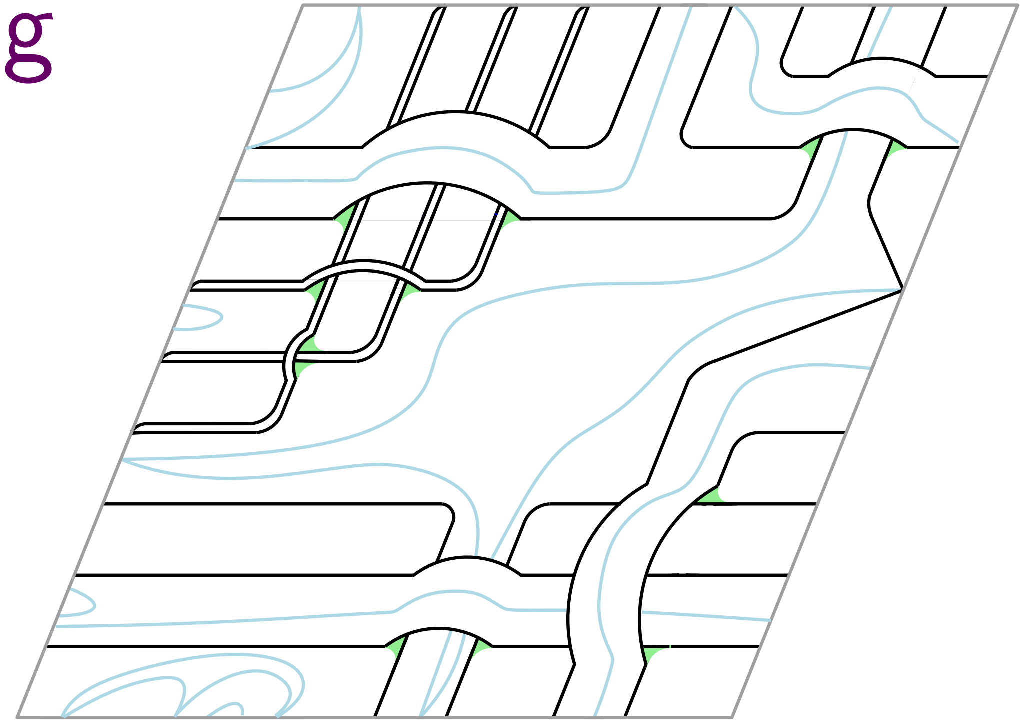

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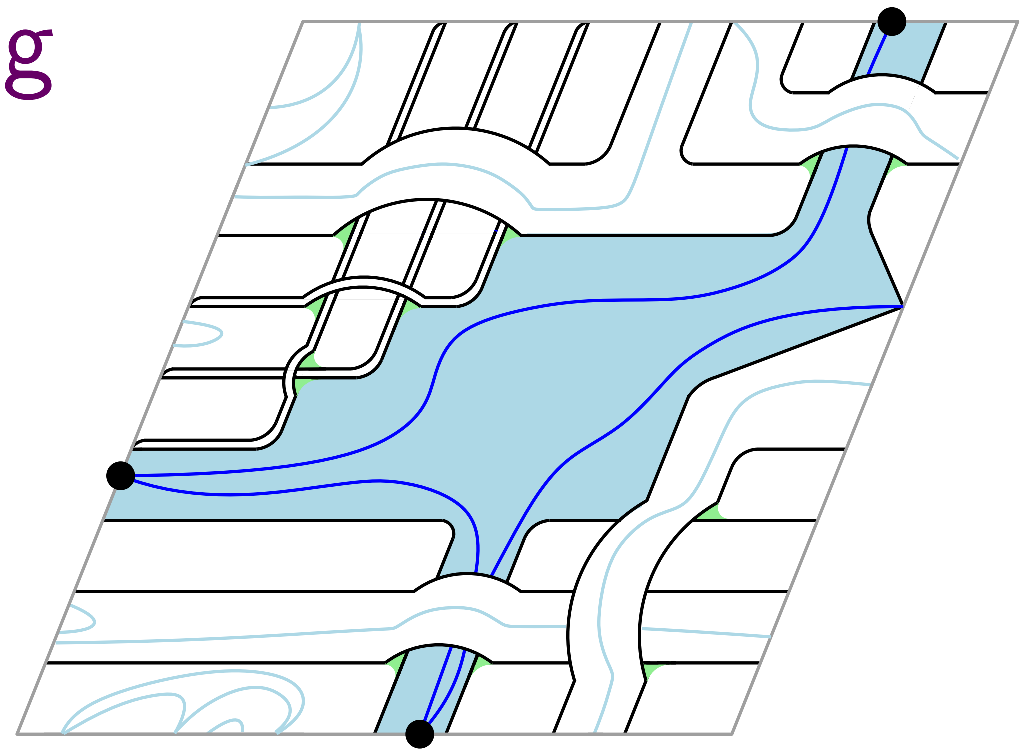

- At most k bundled crossings $\Longrightarrow$ at most $4k$ frame edges.
- We can "lift" the drawing onto a **surface of genus** k
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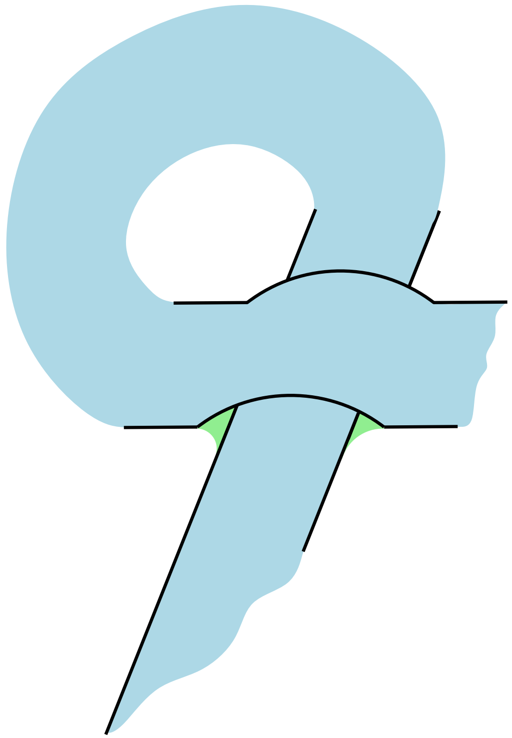
- Other edges/vertices of the graph partitioned into these regions.
- The graph induced by edges inside a single region has a special outerplanar drawing.

Structure of a drawing

What if a region has
a bridge and a tunnel
corresponding to
the same bundled crossing?



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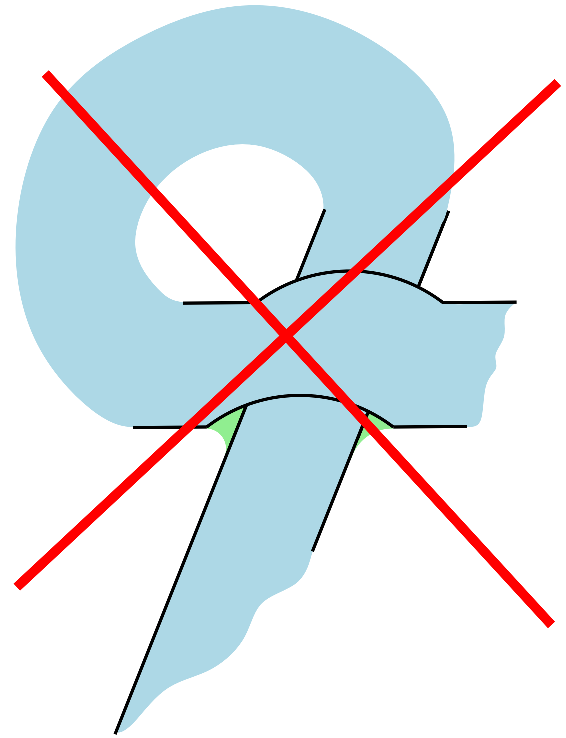

Lem. Each region is a topological disk.

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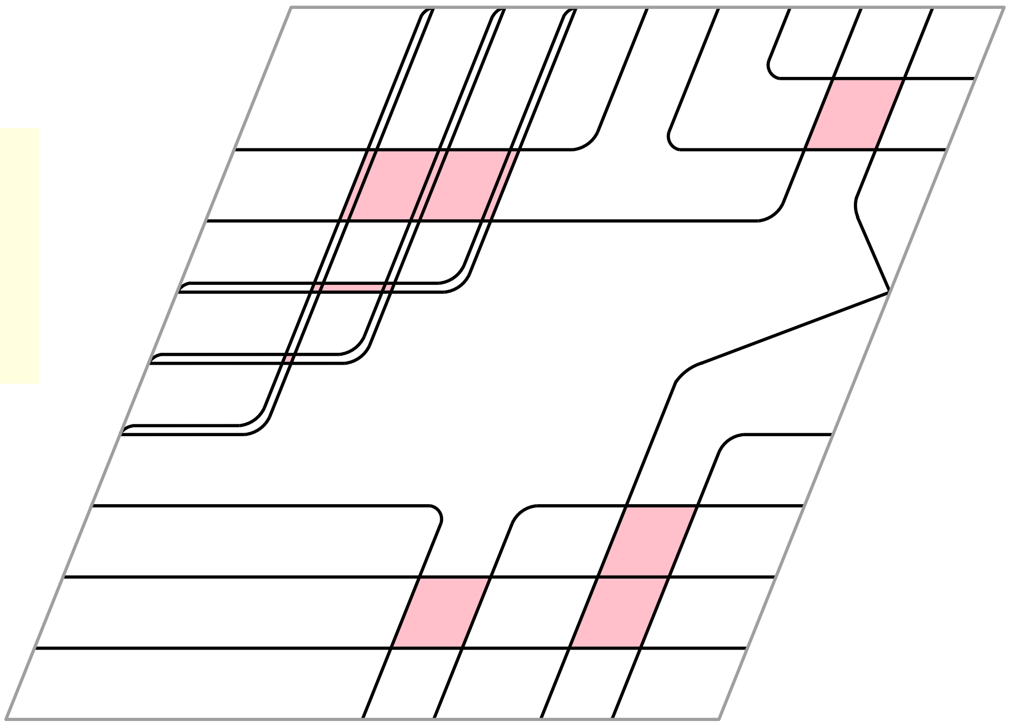
The Algorithm

Thm.
Deciding whether $\mathrm{bc}^{\circ}(G) = k$ is FPT in k.



- Guess the drawing of at most $4k$ frame edges and their bundling.

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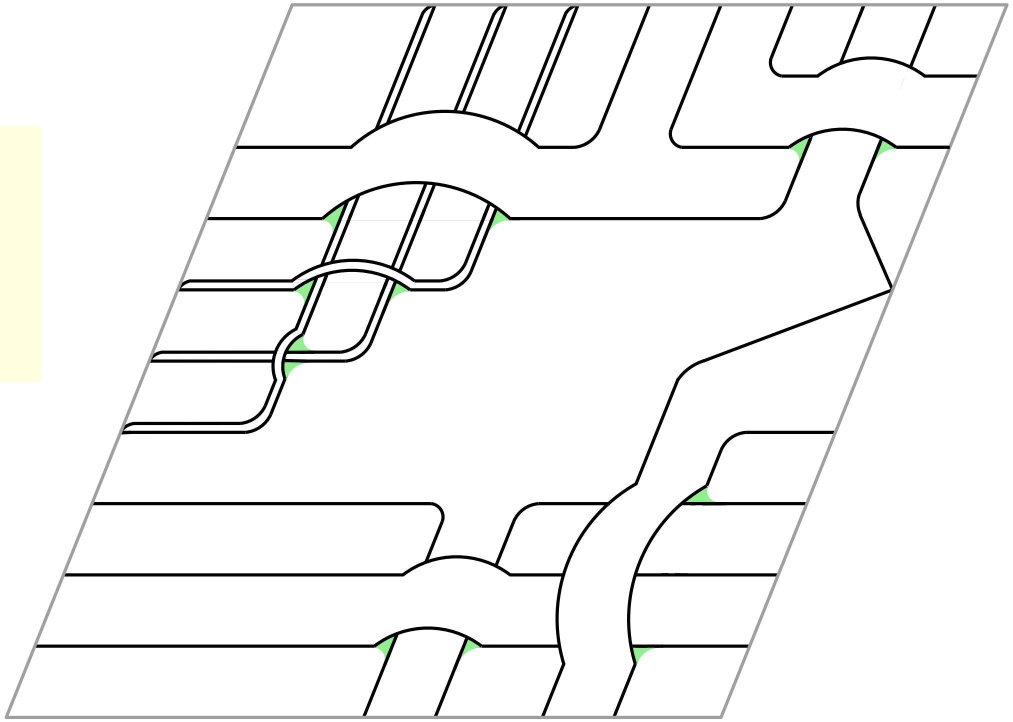

- Guess the drawing of at most $4k$ frame edges and their bundling.
- Construct a **surface of genus** k and a subdivision into **regions**.

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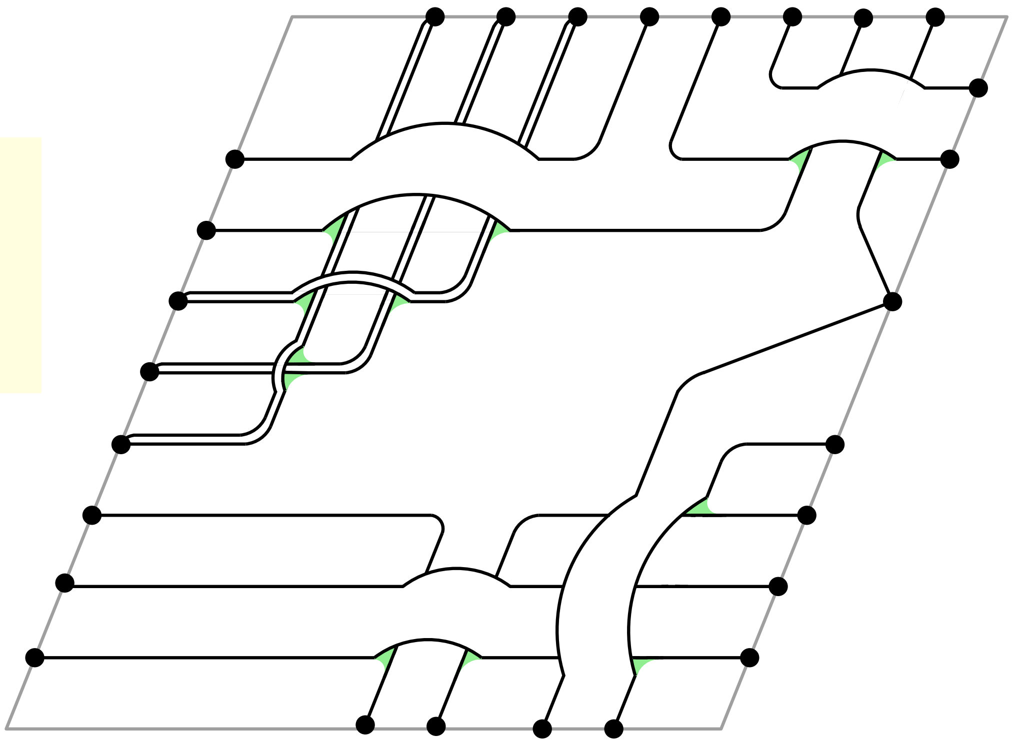
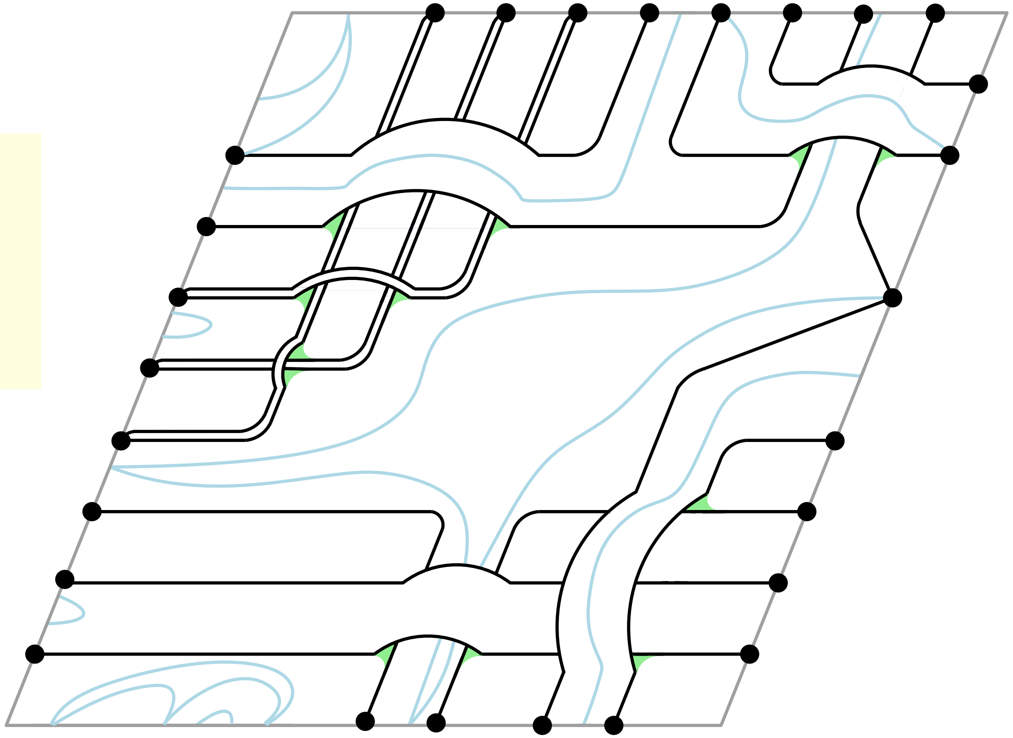


- Guess the drawing of at most $4k$ frame edges and their bundling.
- Construct a **surface of genus** k and a subdivision into **regions**.
  - Map the edges of the graph to the guessed frame edges.

The Algorithm

Thm.
Deciding whether $\mathrm{bc}^\circ(G) = k$ is FPT in k.



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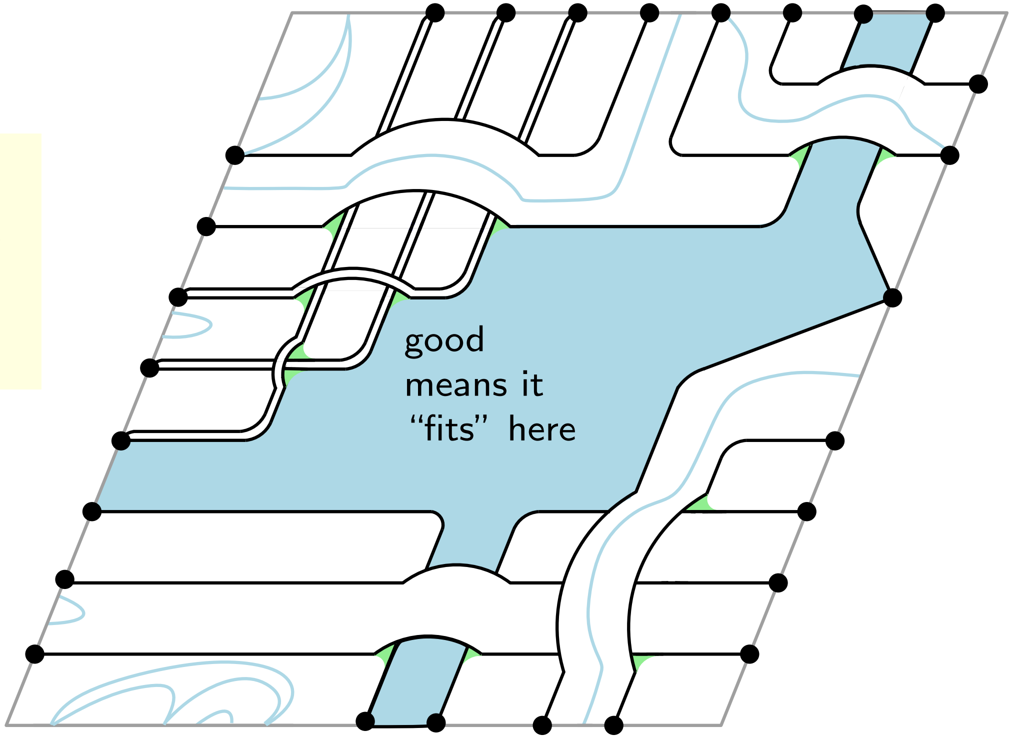

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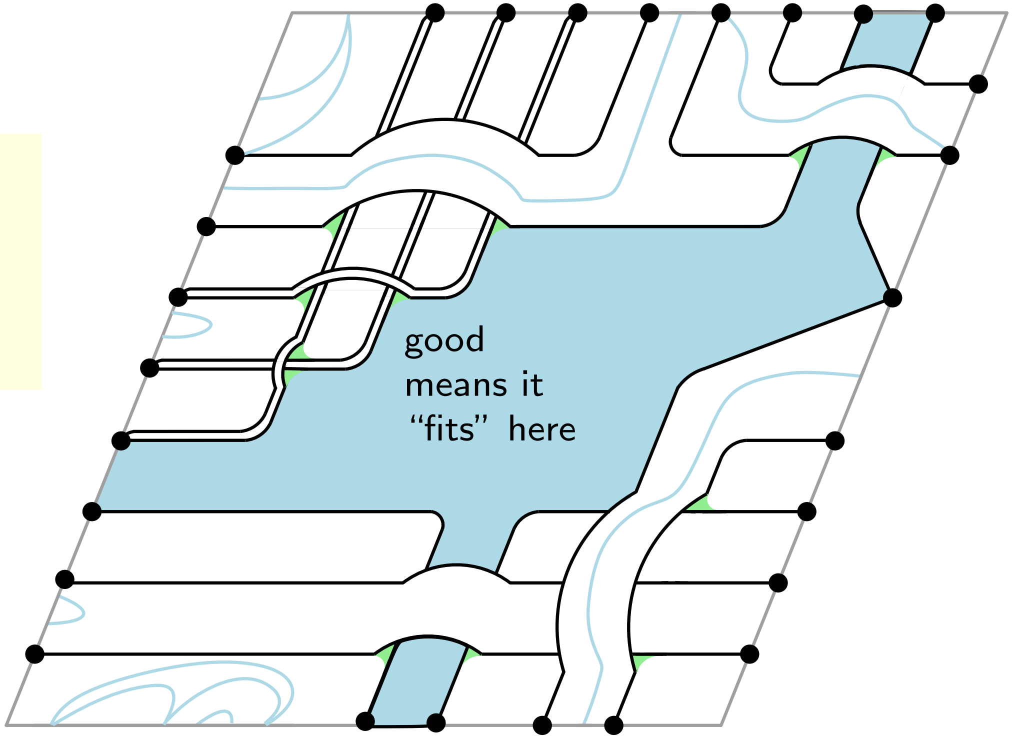

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MSO_2

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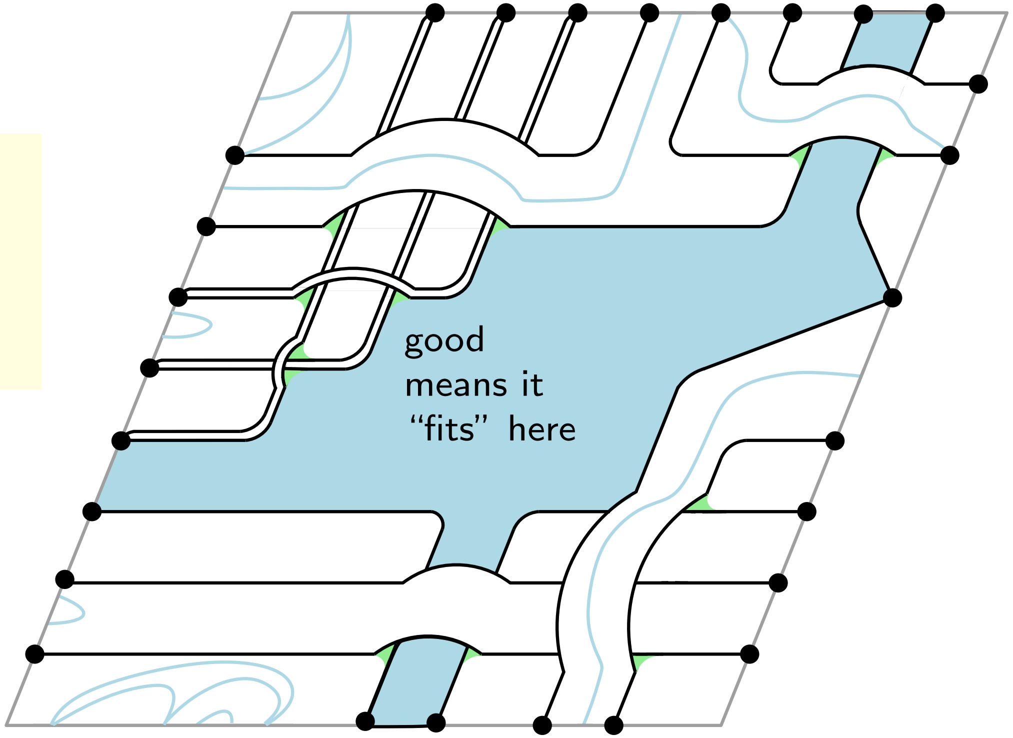

But what is MSO_2 again?

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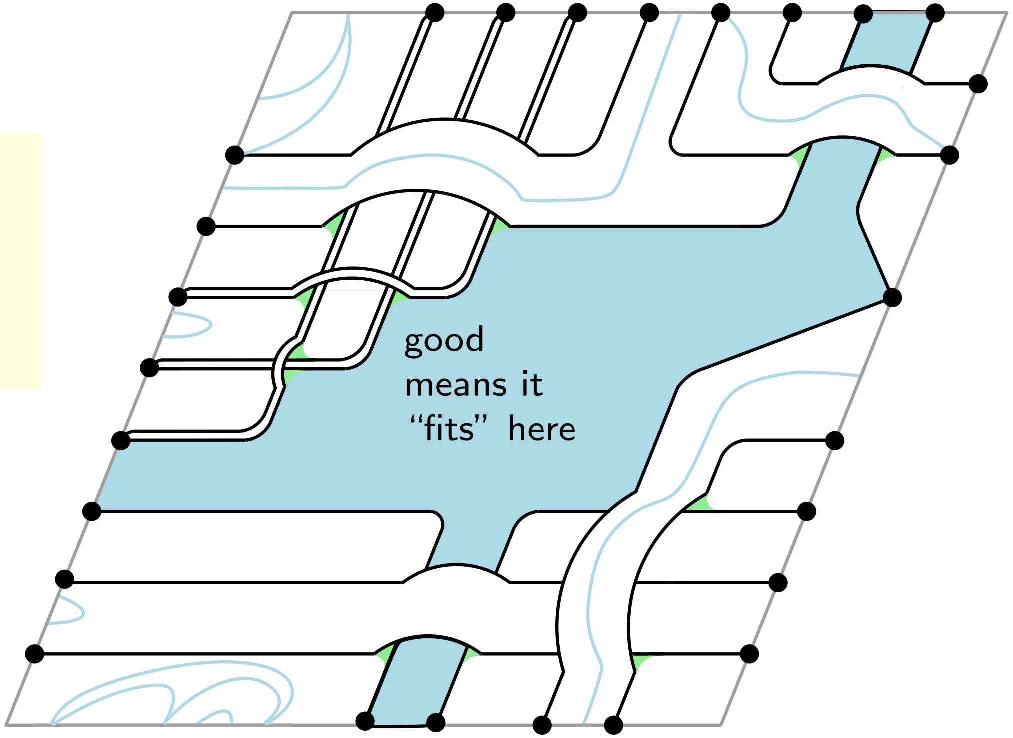

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MSO_2

Monadic Second Order Logic (MSO_2)

$$\textsc{Partition}(E; E_0, \ldots, E_\gamma) =$$

$$(\forall e \in E)\Big[\Big(\bigvee_{i=0}^{\gamma} e \in E_i\Big) \wedge \Big(\bigwedge_{i \neq j} \neg(e \in E_i \wedge e \in E_j)\Big)\Big].$$

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Thm (Courcelle): If a property P is expressed as $\varphi \in MSO_2$, then for every graph G with treewidth at most t, P can be tested in time $O(f(t, |\varphi|)(n + m))$ for a computable function f.

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Since our regions induce outerplanar graphs, we have treewidth at most $8k+2$ where k is the number of bundled crossings.

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This can be stated in MSO_2 via a mechanism of *MSO-definition schemes*, and the *Backwards Translation Theorem* [Courcelle, Engelfriet; 2012]

Monadic Second Order Logic (MSO_2)

Theorem 7.10 (Backwards Translation Theorem) Let $\mathcal{D}$ be a k-copying C_rMS-definition scheme of type $\mathcal{R} \to \mathcal{R}'$ with set of parameters $\mathcal{W}$. Let $\mathcal{X}$ be a finite set of set variables and $\mathcal{Y} = \{y_1, \ldots, y_n\}$ be a set of first-order variables. For every $\beta \in C_rMS(\mathcal{R}', \mathcal{X} \cup \mathcal{Y})$ and $\mathbf{i} \in$ [illegible] one can construct a formula $\beta_{\mathbf{i}}^{\mathcal{D}} \in C_rMS(\mathcal{R}, \mathcal{W} \cup \mathcal{X}^{(k)} \cup \mathcal{Y})$ such that for [illegible] $STR^{c}(\mathcal{R})$, every $\mathcal{W}$-assignment γ, every $\mathcal{X}^{(k)}$-assignment η, and every [illegible]ment μ, all of them in S, we have:

$(S, \gamma \cup \eta \cup \mu) \models \beta_{\mathbf{i}}^{\mathcal{D}}$ if and o[illegible]

$\widehat{\mathcal{D}}(S, \gamma)$ is define[illegible]

$\eta^{[k]} \cup \mu_{\mathbf{i}}$ is a[illegible] assignment in $\widehat{\mathcal{D}}(S, \gamma)$, and

$(\widehat{\mathcal{D}}(S, \gamma)$[illegible] $\models \beta$.

The quantifier-h[illegible] of $\beta_{\mathbf{i}}^{\mathcal{D}}$ is at most $k \cdot qh(\beta) + qh(\mathcal{D}) + 1$. $\square$

It's a bit techn

This can be stated in MSO_2 via a mechanism of *MSO-definition schemes*, and the *Backwards Translation Theorem* [Courcelle, Engelfriet; 2012]

Testing whether bc$^\circ = k$

Thm.
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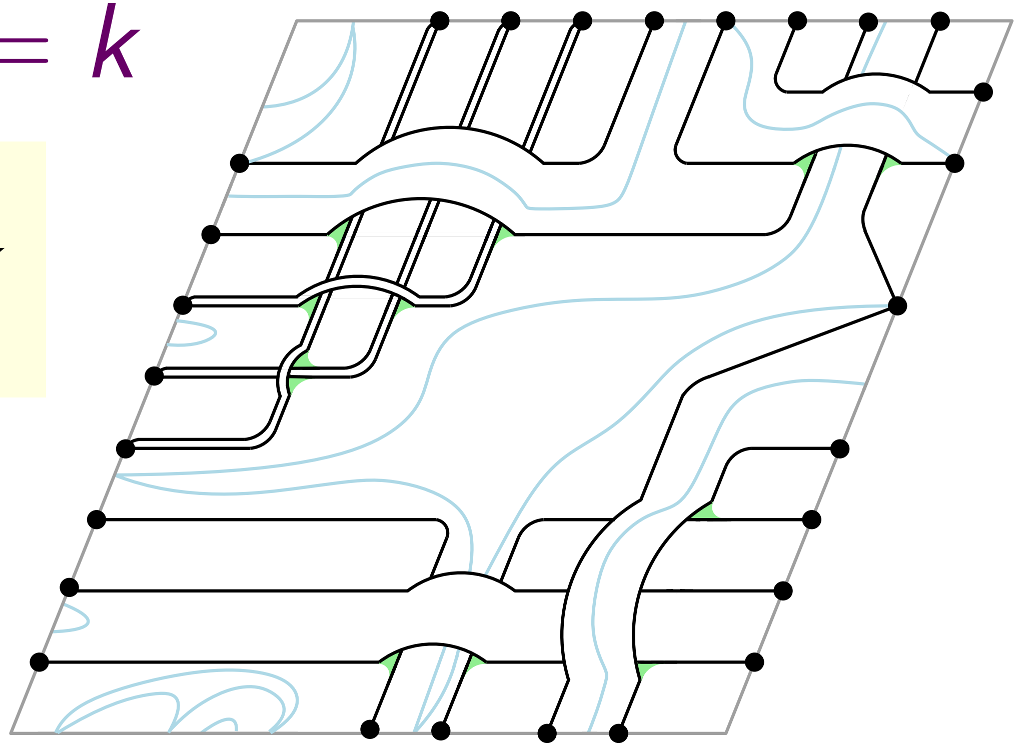

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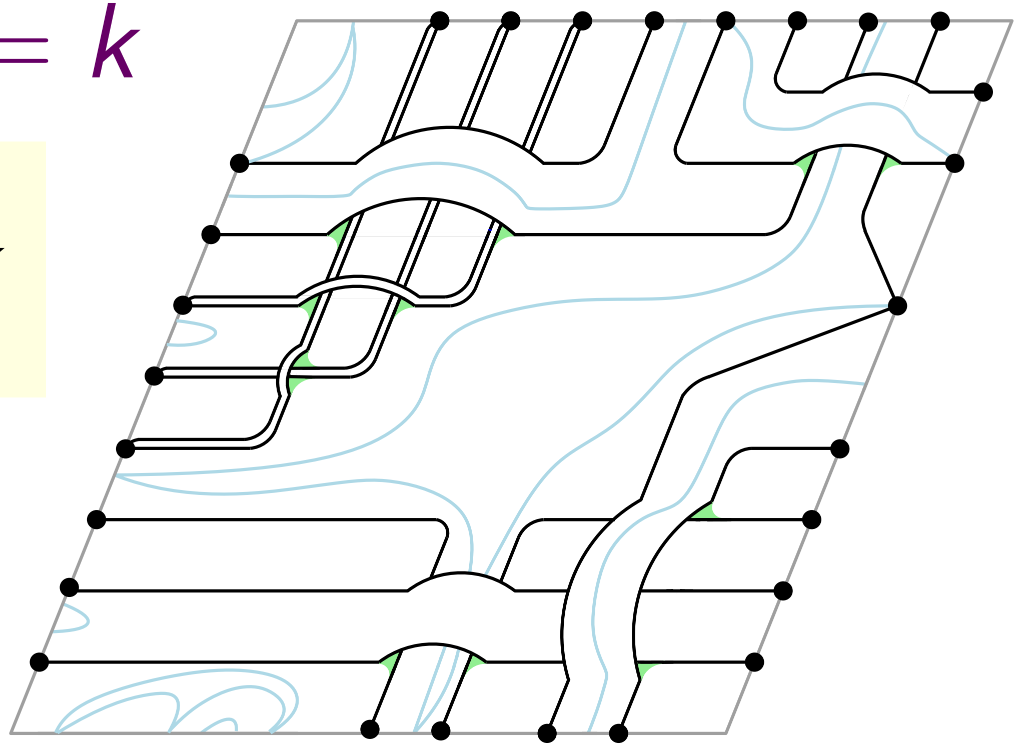

Runtime:

$2^{O(k^2)}$

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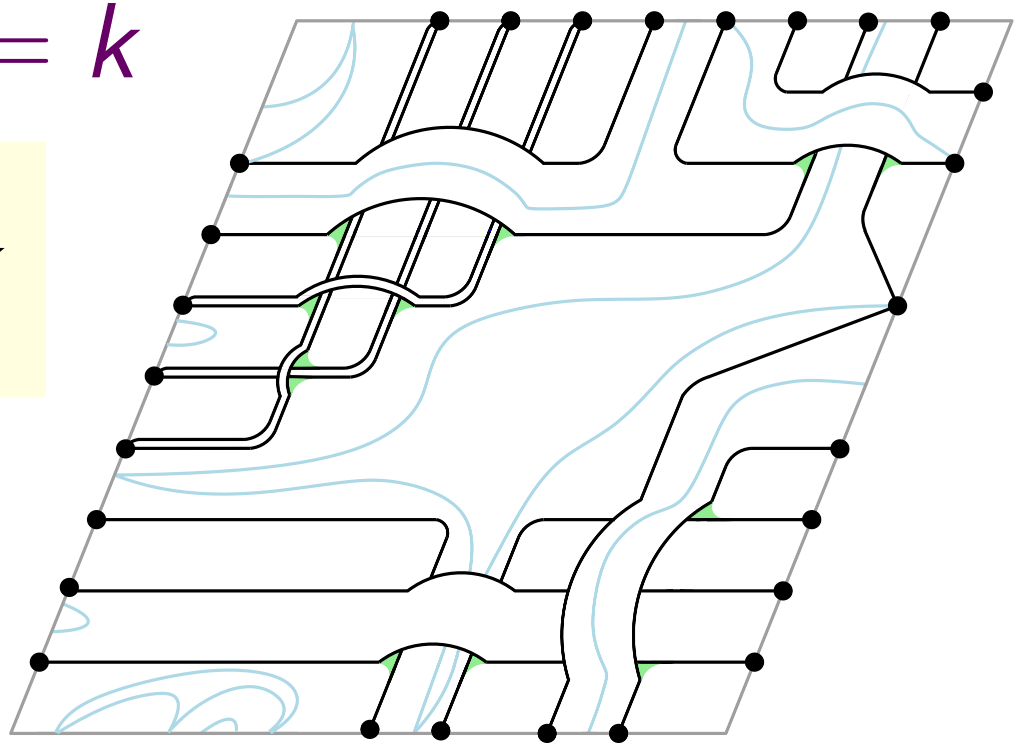

Runtime:

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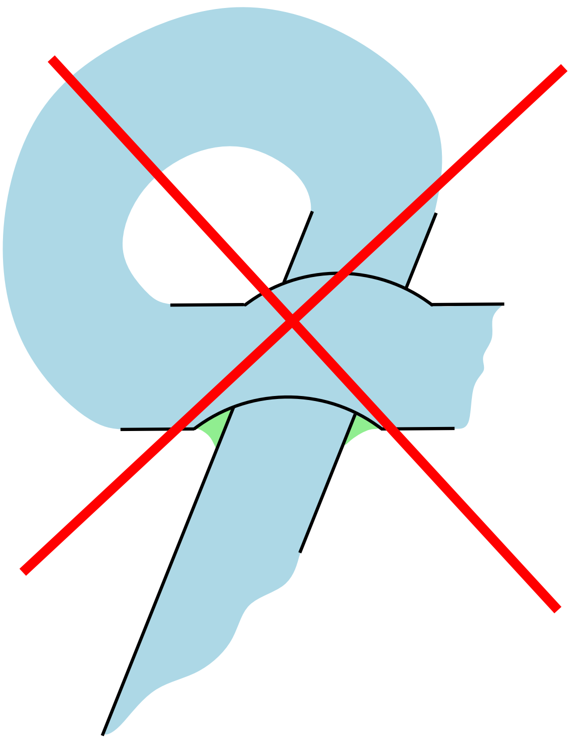

Recall that for correctness of the algorithm we need to show that

Thm. Each region is a topological disk.

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MSO_2

A region is a topological disk

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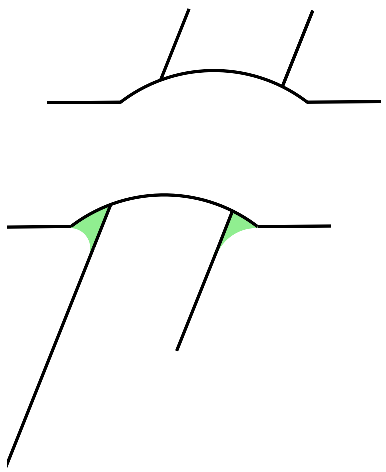
Proof.



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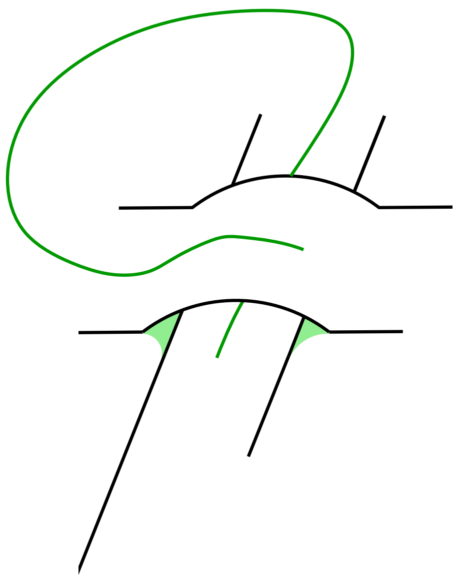

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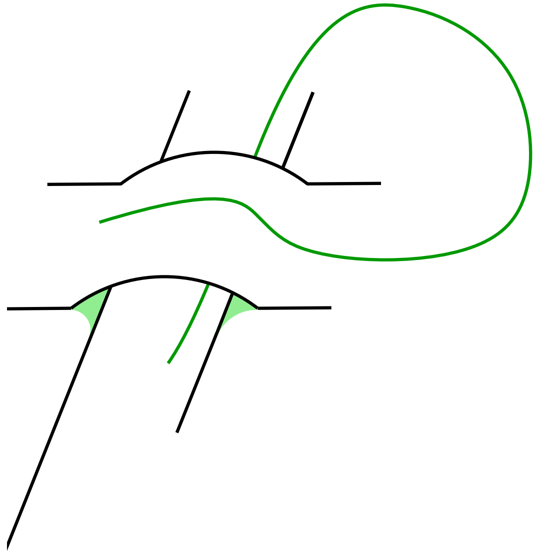

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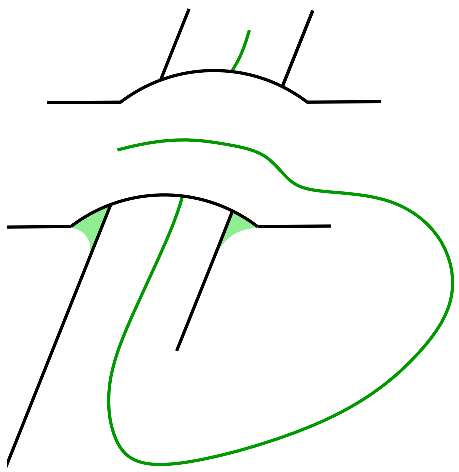

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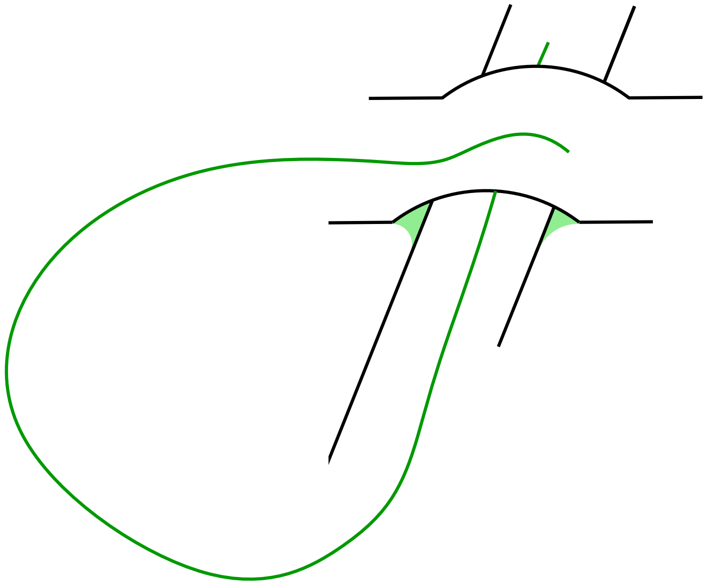

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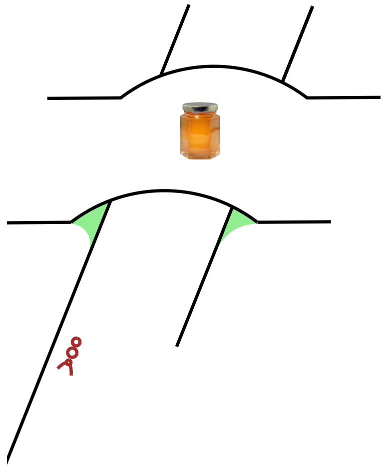

Stick to the right!

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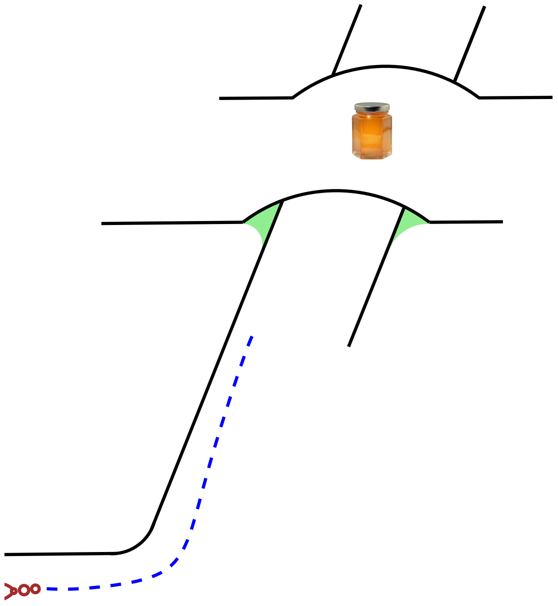

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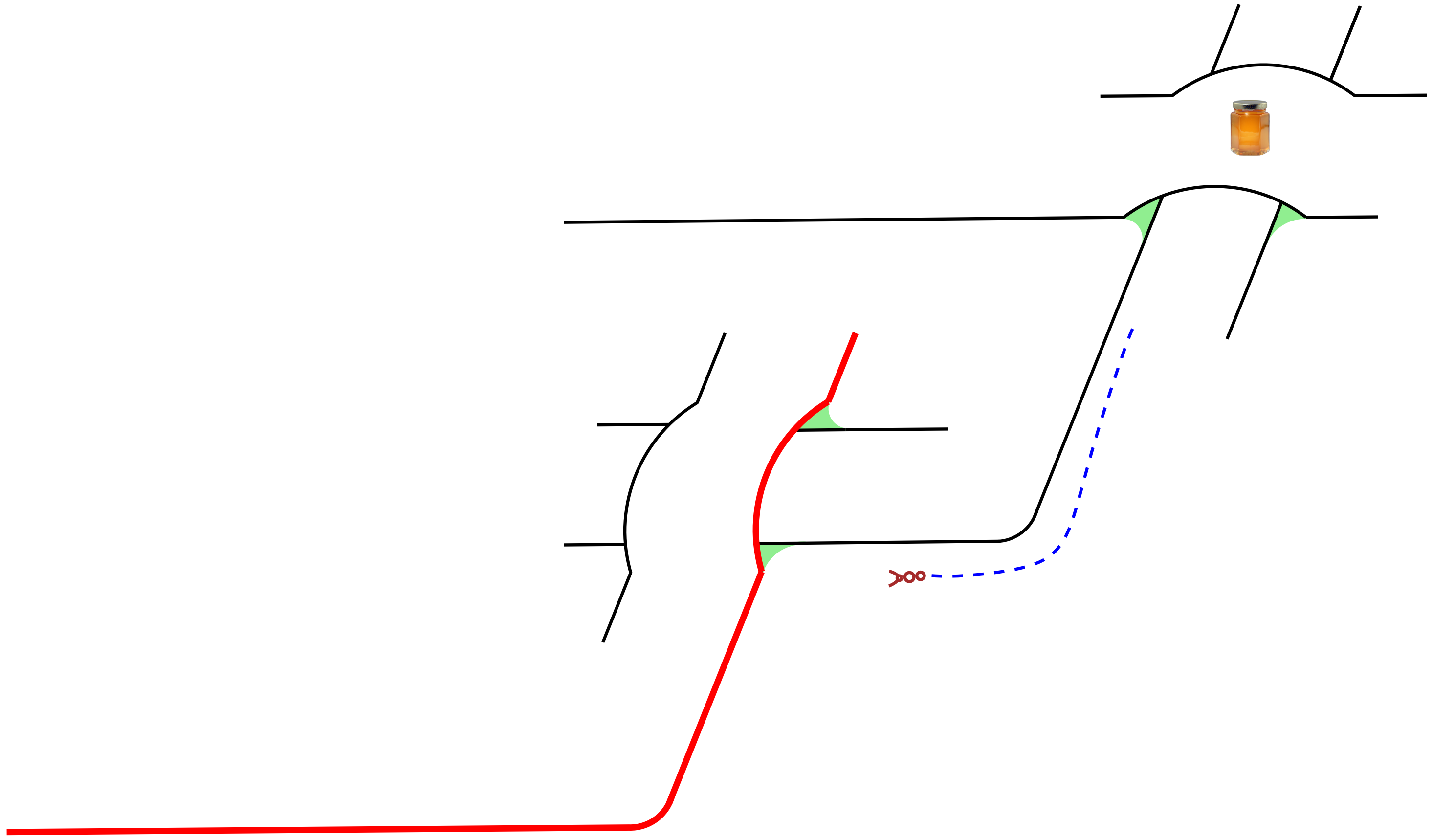

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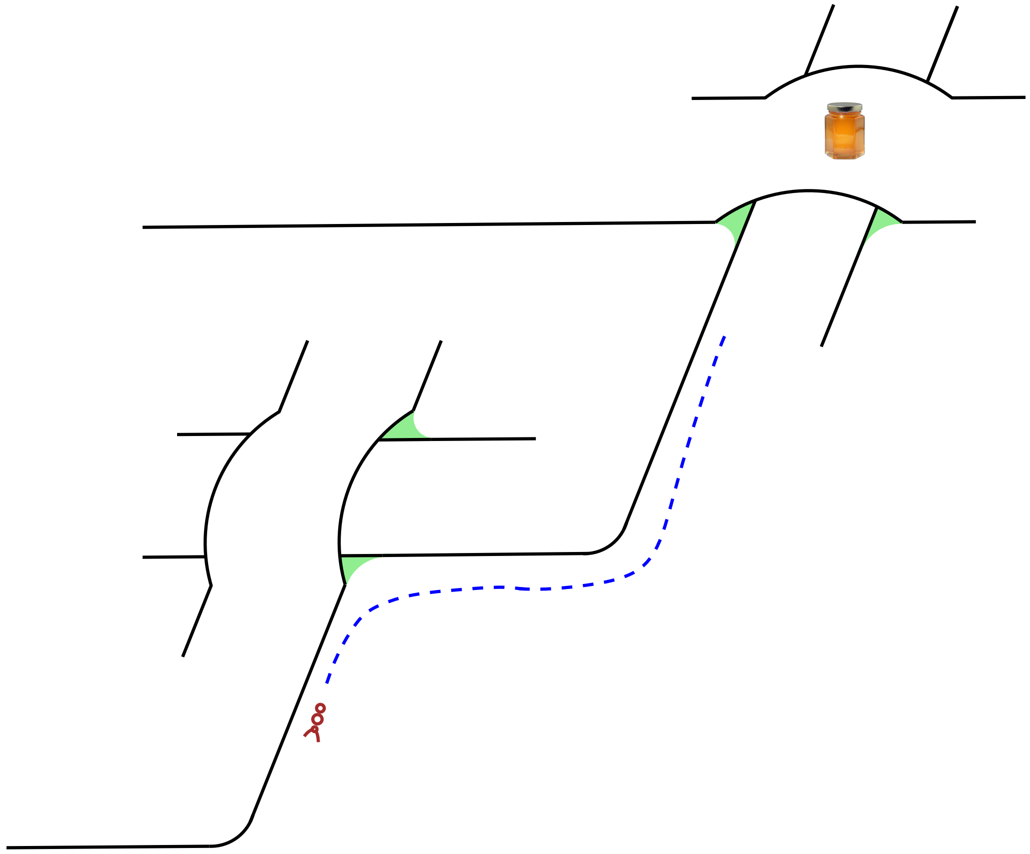

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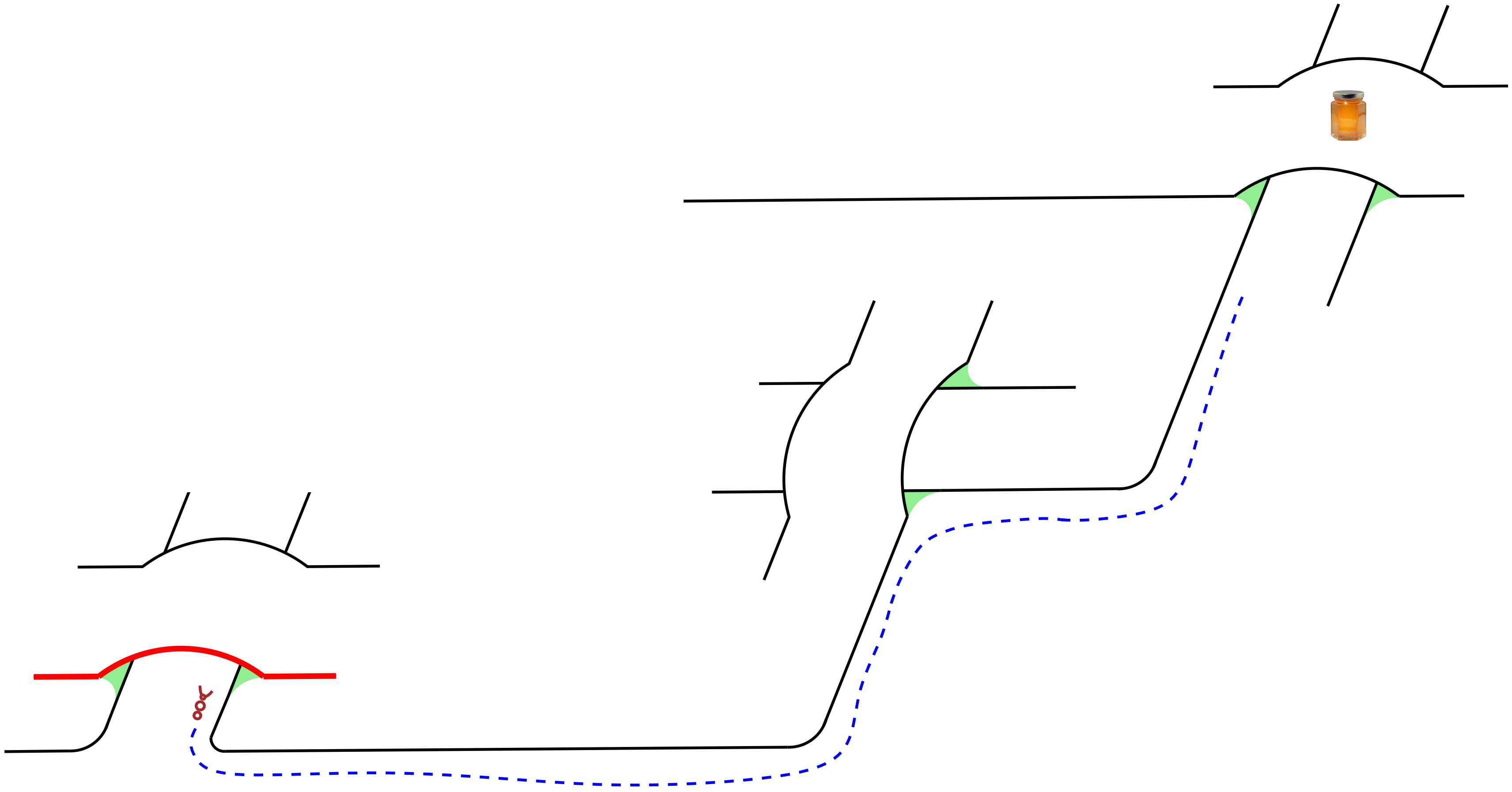

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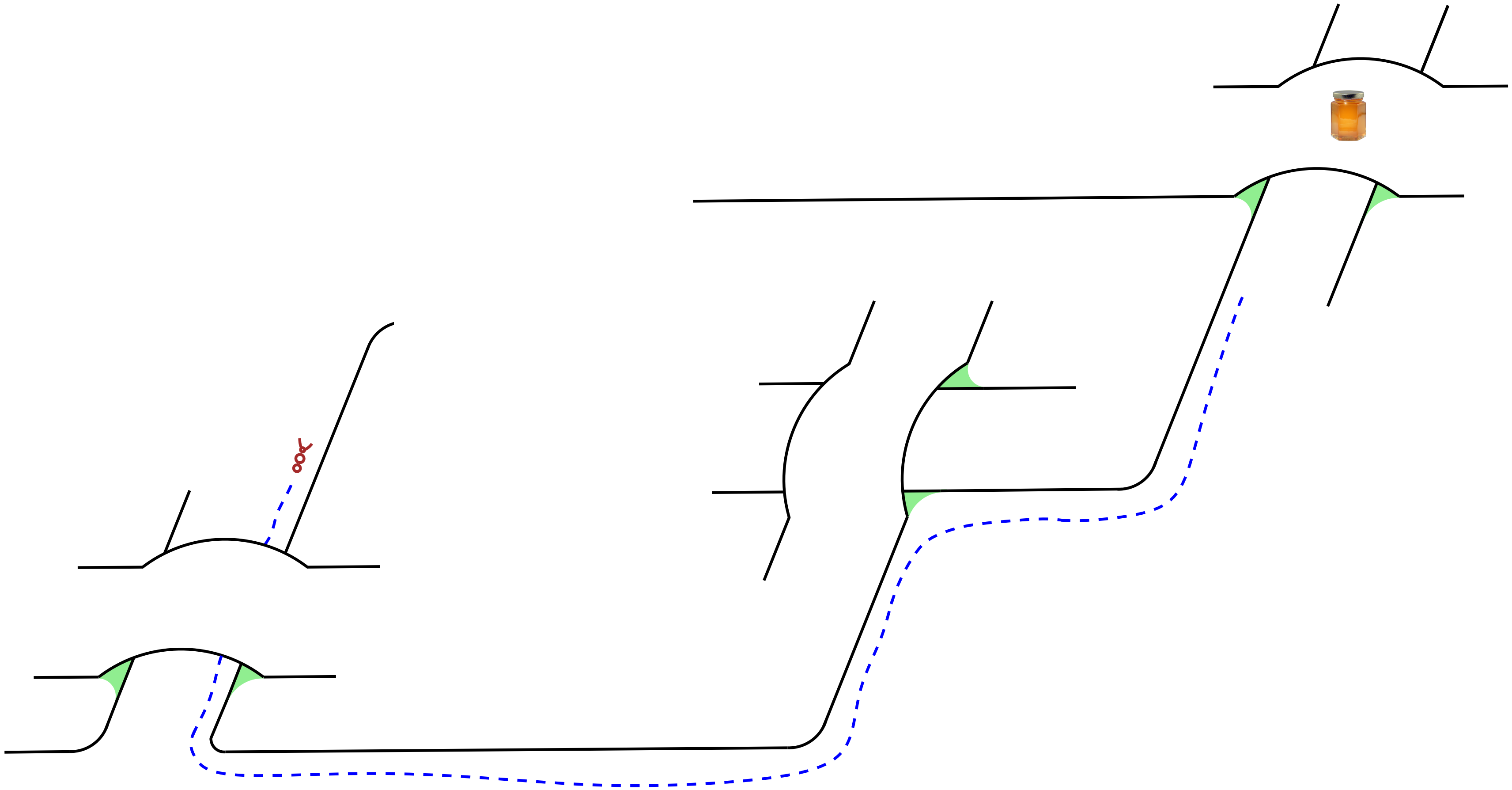

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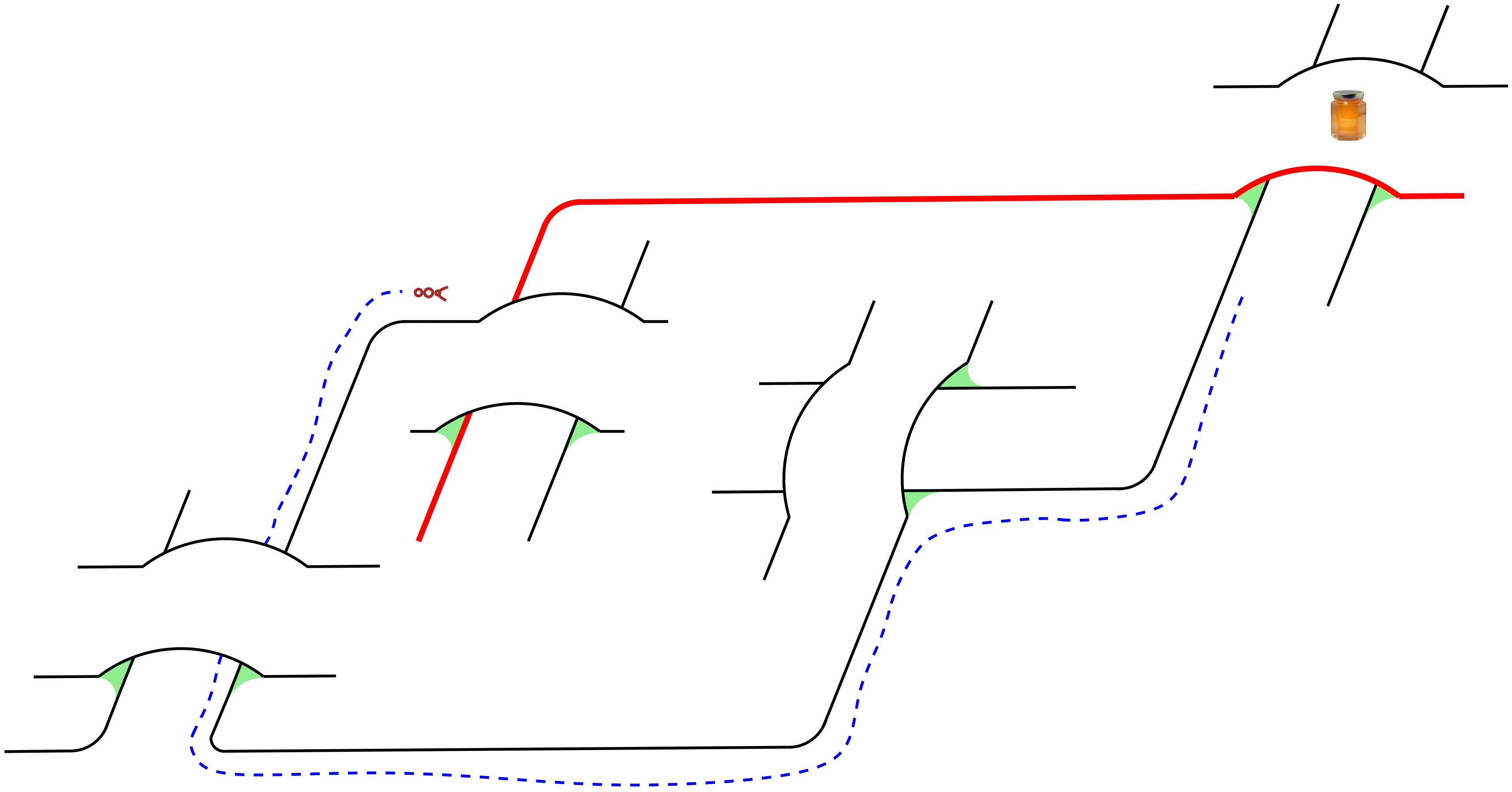

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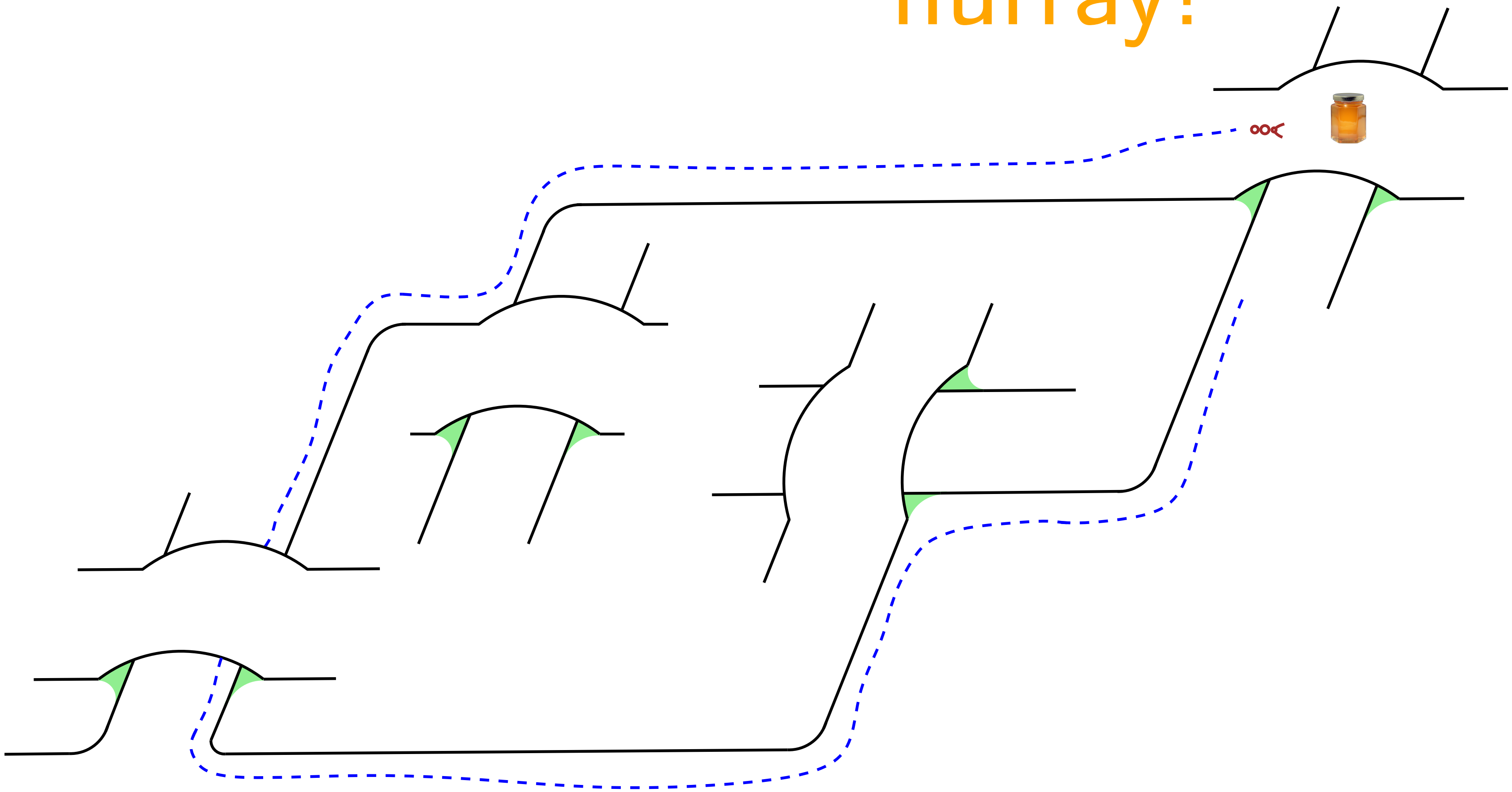
hurray!



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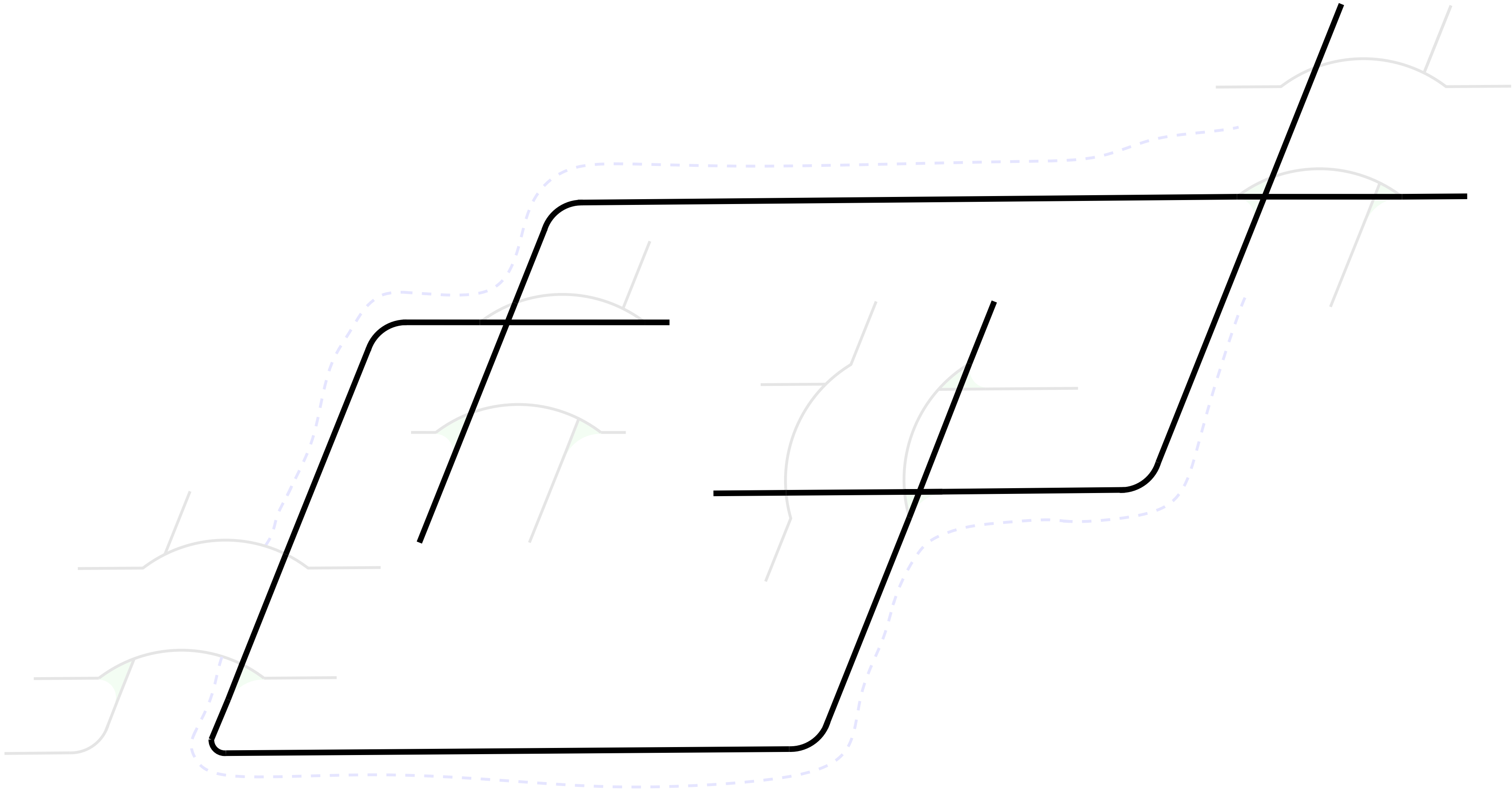

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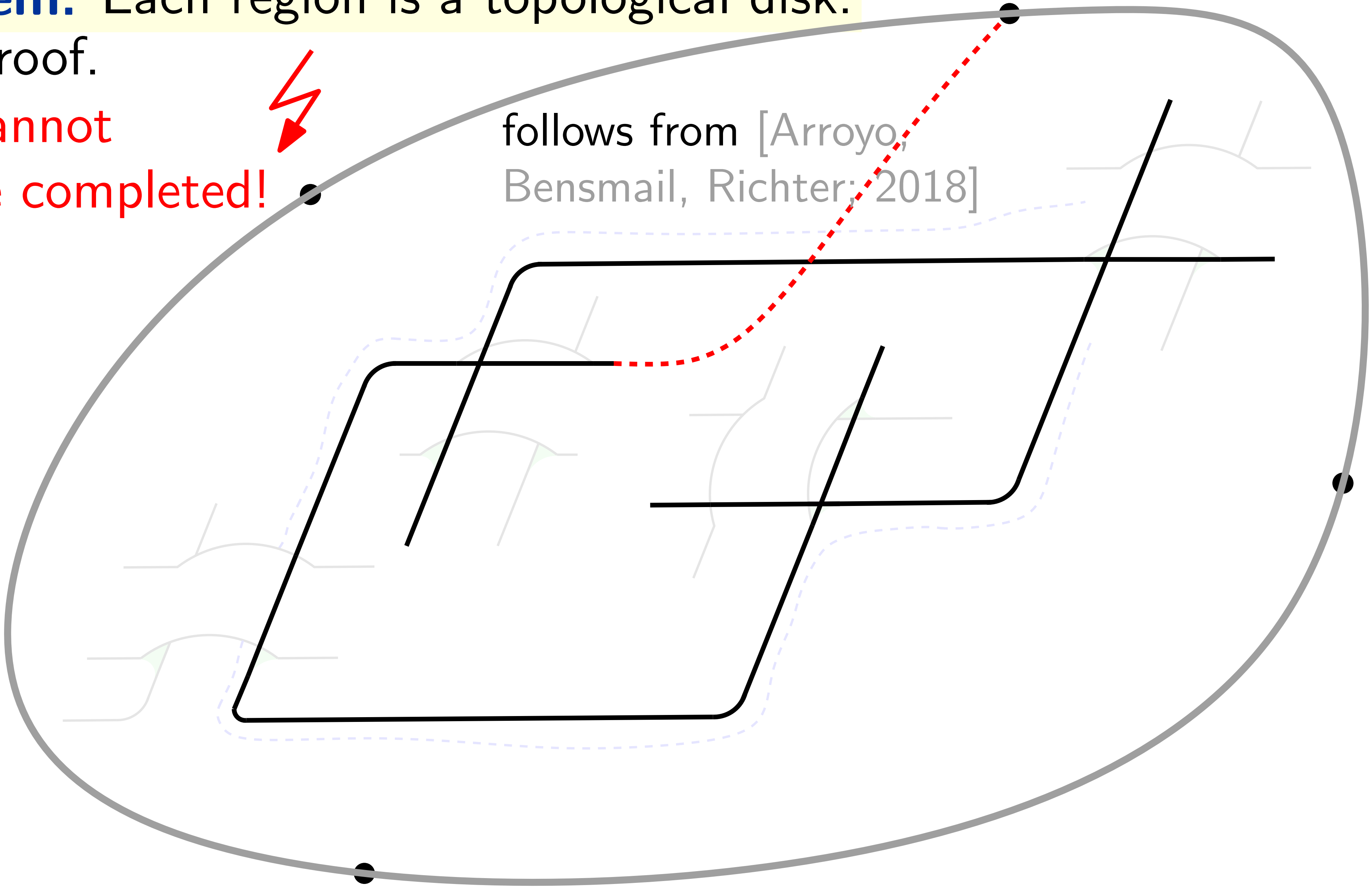

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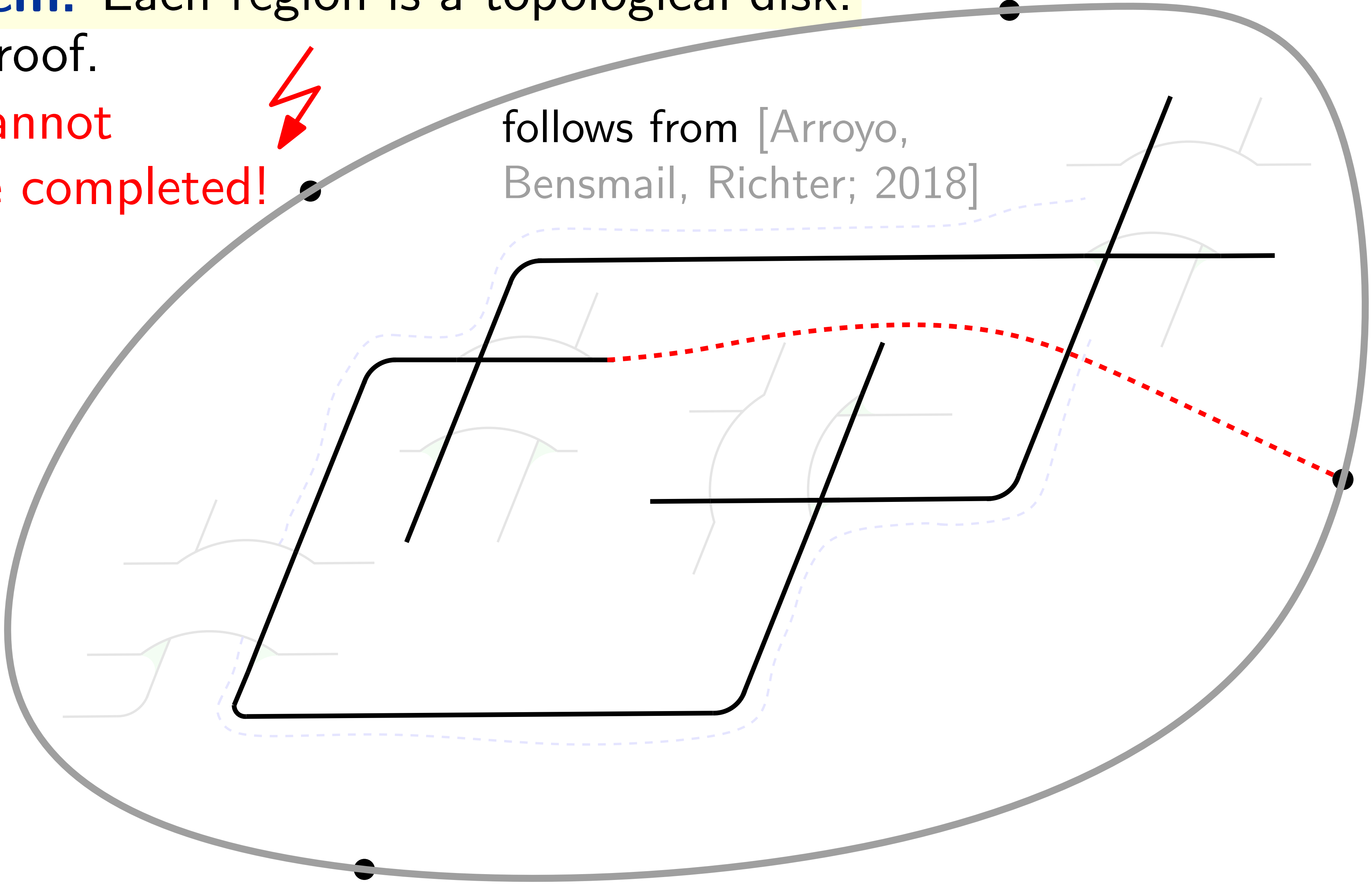

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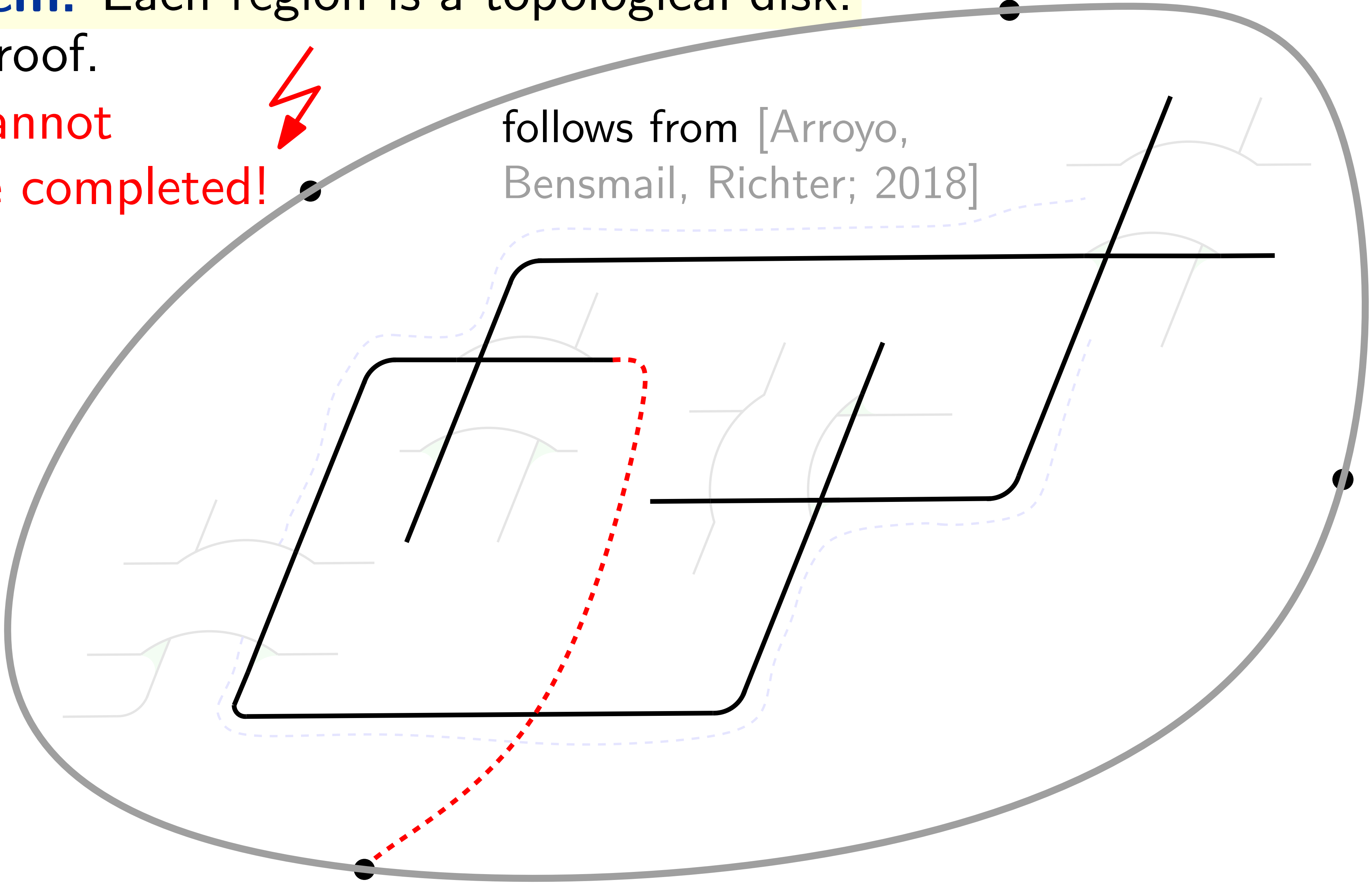

Open Questions

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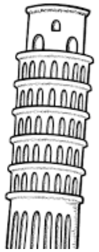 $(k) \times (|V| + |E|)$

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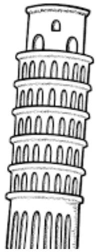 $(k) \times (|V| + |E|)$

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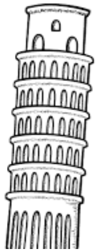 $(k) \times (|V| + |E|)$

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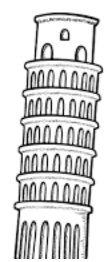 $(k) \times (|V| + |E|)$

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Question 3
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